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A Novel Matching of MR Images Using Gabor Wavelets

Mehmet Ali Akinlar, Muhammet Kurulay[1], Aydin Secer[1] and Mehmet Celenk[2]

Bilecik Seyh Edebali University, Bilecik, 11210, [1]Yildiz Technical University, Istanbul, Turkey, [2]Ohio University, Ohio, USA

Abstract

This study introduces a hybrid method for deformable matching of Magnetic resonance (MR) images by utilizing the advantages of both wavelets and variational calculus. Image matching problem is expressed as an optimal control problem and discretization of the resulting Euler-Lagrange equations is written in terms of the system of linear equations in the form of $Au = f$, where u is the image displacement field. Implementation of the algorithm exploits Gabor wavelet energy maps of MR images. The proposed algorithm provides an efficient MR matching technique. Experimental results proved that the method can match MR images better than the only variational or only wavelet-based methods.

Keywords

Gabor wavelets, Image matching, Magnetic resonance imaging.

1. Introduction

Image matching (sometimes known as image registration or alignment) can be defined as the process of establishing pixel-by-pixel correspondences between two (or more) images. The images could be of the same or different objects and of different imaging modalities (Magnetic resonance imaging (MRI), X-ray, CT, PET, SPECT, tomography, and so on) and possibly be taken at different distances, angles, and times. In this paper, we focus on MRI which is a significant medical imaging modality used to analyze internal structures of the body components in detail. Given a reference image $R(x)$ and a template image $T(x)$, the main idea behind image-matching paradigm is to find a reasonable transformation such that the transformed template image becomes similar to the reference image. Object and motion tracking, detecting tumors, locating diseased areas, fusion of different image modalities, and remote sensing are some well-known applications of image matching. Image matching is an important and challenging subject that usually involves high storage requirements, high CPU costs, noisy and distorted data, and occlusion. So far, a general theory for image matching has yet to be established. Each application venue has developed its own approaches and implementations. As a result, a single standard method for image matching has not emerged. Therefore, finding reliable and efficient image matching techniques along with fast implementation methods are significantly important and active research areas. Some of the well-known image matching algorithms can be seen at [1-7] and in the references therein. Further imaging applications where the image matching plays a significant role might be seen at [8-10].

Mathematical models and wavelets might be quite useful in the image processing problems. The main purpose of this paper is to present some models for fast and efficient matching of MR images. In order to succeed this, a hybrid method which utilizes the advantages of both wavelets and variational calculus techniques is proposed. Energies of MR images are expressed in terms of Gabor wavelet transform. Image matching problem is expressed as an optimal control problem and the discretization of the resulting equations, namely Euler-Lagrange equations is expressed in terms of a system of linear equations, $Au = f$, where u is the image displacement field. Using the fact that two-dimensional images can be considered as matrix operators, the present method utilizes the advantages of both wavelets and variational deformable image matching.

Organization of this paper is as follows: Section 2 introduces the concept of wavelet, and particularly describes the Gabor wavelet. Main results are given in Section 3 where we write the image matching algorithm as an optimal control problem. Using finite difference method, we point that the resulting Euler-Lagrange equations of this optimization problem can be written as $Au = f$, where u is the image displacement field. The clear connection between the wavelet methods and variational methods is established in the implementation section. In the final section, some experimental results are provided as an application of the proposed algorithm. Experimental results proved that the present method can match MR images quite fast and efficiently. We compare the present method with some other related methods employing variational or wavelet-based methods.

2. Gabor Wavelets

A function $\psi \in L_2(\mathrm{R})$ is said to be a wavelet if the family of functions $\psi_{j,k}(t)$ defined by

$$\psi_{j,k}(t) = 2^{j/2}\psi(2^j t - k) \tag{1}$$

where, j and k are arbitrary integers, is an orthonormal basis in the Hilbert space L_2 (R).

Wavelet coefficients of a function $f \in L_2(\mathrm{R})$, is denoted by

$$d_{j,k} = \int_{\mathrm{R}} f(t)\psi_{j,k}(t)dt \tag{2}$$

The series

$$\sum_{j,k\in Z} \psi_{j,k}(t)\int_{\mathrm{R}} f(t)\psi_{j,k}(t)dt \tag{3}$$

is called the wavelet series of $f \in L_2$ (R). The expression

$$f = \sum_{j,k\in Z} \psi_{j,k}(t)\int_{\mathrm{R}} f(t)\psi_{j,k}(t)dt \tag{4}$$

is called the wavelet representation of f. Detailed information about wavelets and their applications to image processing problems can be seen, for example, at [11].

The function defined by the equation

$$\psi(x) = w^{-1/2}e^{-\pi(x/w)2}\, e^{i2\pi vx/w} \tag{5}$$

satisfies (1) where w and v are width and frequency parameters, respectively. The wavelet given by (5) is known as Gabor wavelet. In the next section, we present a hybrid method for deformable matching of MR images, which takes the advantages of both wavelets and techniques of variational calculus.

3. Variational Formulation of Deformable Image Matching

The state of the art of the image registration problem can be expressed in the following way. Assume that both the template T and reference R images are defined on the same domain Ω. Then, the image registration problem can be formulated as the optimization problem

$$\min_{\varphi\in\Gamma} \mathrm{J}[R,T;\varphi_u] \tag{6}$$

for the functional

$$\mathrm{J}[R,T;\varphi_u] = C_{sim}[R,T;\varphi_u] + \lambda C_{reg}[u], \tag{7}$$

where, C_{sim} $[R, T;\varphi_u]$ denotes a similarity measure between the template image T and the reference image R, $\phi_u(\mathrm{x}) := \mathrm{x} + \mathrm{u}(\mathrm{x})$ is the deformation field, u is displacement field, Γ is the set of all possible admissible transformations, C_{reg} $[u]$ is a regularization term, and λ is a regularization constant. Because reference and template images are obtained from different distances, angles, times, and sometimes even by different individuals, a deformation field may occur between these images. A deformation field is a vector field that maps pixels of reference image to the corresponding ones of the template image. One of the major goals of this paper is to compute the deformation field in a systematic way.

We choose the L^2-norm type similarity measure defined as

$$C_{sim}[R(\mathrm{x}),T(\mathrm{x});\varphi(\mathrm{x})] = \frac{\int_\Omega (\mathrm{T}(\mathrm{x}+\mathrm{u}(\mathrm{x})) - \mathrm{R}(\mathrm{x}))^2\, d\mathrm{x}}{2\| T(\mathrm{x})\|_{L_2} \| R(\mathrm{x})\|_{L_2}}. \tag{8}$$

Let us notice that other similarity measures such as the one described at [12] can be selected depending on the problem. We choose the similarity measure (8) only for the convenience in computations.

Image registration is an ill-posed optimal control problem. In order to overcome the ill-posedness of the optimization problem (6) and to assure smooth solutions, we introduce the additional regularization terms. The main idea behind adding a regularization term is to smoothen the problem with respect to both the functional and the solution, so that well-posedness is assured and efficient computational methods can be developed to determine minimizers. In this paper, we use a regularization term given by

$$C_{reg}[u] := \int_\Omega \sqrt{|\nabla u|^2 + \beta} \tag{9}$$

where, $\beta \geq 0$ is an arbitrarily small parameter to assure the smoothness of the solutions. The well-posedness of these types of problems with $\beta \to 0$ has been discussed at [4]. With these similarity measure and regularization terms, the functional in (7) is strictly convex with a unique minimizer.

For the convenience of the computations, we define T_u (x): $= To\varphi_u(\mathrm{x}) = T$ (x + u (x)). Writing the regularization term (9) in the functional (7) and taking into account the boundary conditions, we can write the functional (7) as

$$\mathrm{J} = \frac{\int_\Omega (T(\mathrm{x}+u(\mathrm{x})) - R(\mathrm{x}))^2 d\mathrm{x}}{2\|T(\mathrm{x})\|_{L_2}\|R(\mathrm{x})\|_{L_2}} - \lambda\int_\Omega \sqrt{|\nabla u(\mathrm{x})|^2 + \beta}\, d\mathrm{x} + \int_{\partial\Omega} \frac{1}{|\nabla u(\mathrm{x})|}\frac{\partial u(\mathrm{x})}{\partial \mathrm{n}} d\mathrm{H}^1 d\mathrm{x}, \tag{10}$$

where, n denotes the outer normal along the boundary ∂Ω, and $d\mathrm{H}^1$ is the 1 – dimensional Hausdorff measure supported on ∂Ω. Similar types of optimal control problems with their computational solutions might be seen at [5].

Using techniques of calculus of variations, the corresponding Euler-Lagrange equations for (10) is given by

$$\frac{\nabla T_u(\mathrm{x})(T_u(\mathrm{x})-R(\mathrm{x}))}{\|T(\mathrm{x})\|_{L_2}\|R(\mathrm{x})\|_{L_2}}-\lambda\nabla\cdot\left(\frac{\nabla u(\mathrm{x})}{\sqrt{|\nabla u(\mathrm{x})|^2+\beta}}\right)=0 \qquad (11)$$

with homogeneous Neumann boundary conditions along $\partial\Omega$. Using finite-differences numerical method, (11) can be written in terms of a system of linear equations as $Au = f$.

It is known that wavelet transforms might be used efficiently in obtaining numerical solutions of some linear differential equations. The matrix operator A in solving a system of linear equations given in the form of $Au = f$, might be considered as a two-dimensional image. By exploiting energy-compaction feature of two-dimensional wavelet transforms, we can think of a large amount of wavelet coefficients to be small and negligible. In order to solve the linear system of equations given by $Au = f$, with wavelets, first we take the two-dimensional wavelet transform of the coefficient matrix, A, as $\tilde{A} = WAW^T$, and the vector b as $\tilde{b} = Wb$, where W represents the kernel of the wavelet transform operator. Next, we solve for $\tilde{A}\tilde{u} = \tilde{b}$, where $\tilde{u}$ represents the wavelet transform of the solution vector u. In order to get the solution u, final inverse of the wavelet transform $u=W^T\tilde{u}$ is taken. In order to figure out better this, nice combination of variational methods and wavelets we refer the interested reader to [13] where the authors present a similar method involving techniques of variational calculus and wavelets for image denoising problems. In this paper, [13] the authors set up a clear connection between a variational problem and its representation in the form of $Au = f$.

4. Experimental Results

In this section, we demonstrate the registration of brain MR images in the size of 65 × 65 as an application of the present method. The template, reference, and matched images are shown in Figure 1. Duration of the registration is about 1 minute, which is quite fast for medical image matching. We applied the presented method to some other brain MR images and obtained the similar results.

5. Comparison with Some other Related Methods

In this paper, we present a hybrid image matching algorithm which employs both the techniques of variational calculus and wavelets. In this Section, first we briefly overview some related methods and secondly compare the method with those related methods.

In a recent paper, [6] the authors presented a method for integration of 3–D medical data by utilizing the advantages of 3–D multi-resolution analysis and techniques of variational calculus. They first expressed the data integration problem as a variational optimal control problem where the displacement field was written in terms of wavelet expansions and secondly they wrote the components of the displacement field in terms of wavelet coefficients. The authors solved this optimization problem with a block-wise descent algorithm and demonstrated the application of the method by registering 3–D brain MR images in the size of 257 × 257 × 65. Duration of the medical data integration process was about 2 minutes and the registered image seems has features of both reference and template image. Detailed information about this method can be seen at [6].

In another paper, [5] the authors introduce several mathematical image registration models employing some curvature-driven diffusion-based techniques, in particular, Perona–Malik, anisotropic diffusion, mean curvature motion (MCM), affine invariant MCM (AIMCM). Adopting the steepest-descent marching with an artificial time t, Euler-Lagrange (EL) equations with homogeneous Neumann boundary conditions are obtained. These EL equations are approximately solved by the explicit Petrov-Galerkin scheme. The method is applied to the registration of brain MR images of size 257 × 257. Computational results indicate that all these regularization terms produce similarly good registration quality, but the cost associated with the AIMCM approach is, on the average, less than that for the others. Duration of the registration with each model was around 1 to 3 minutes depending on the diffusion term and the quality of the registered images was quite good as well.

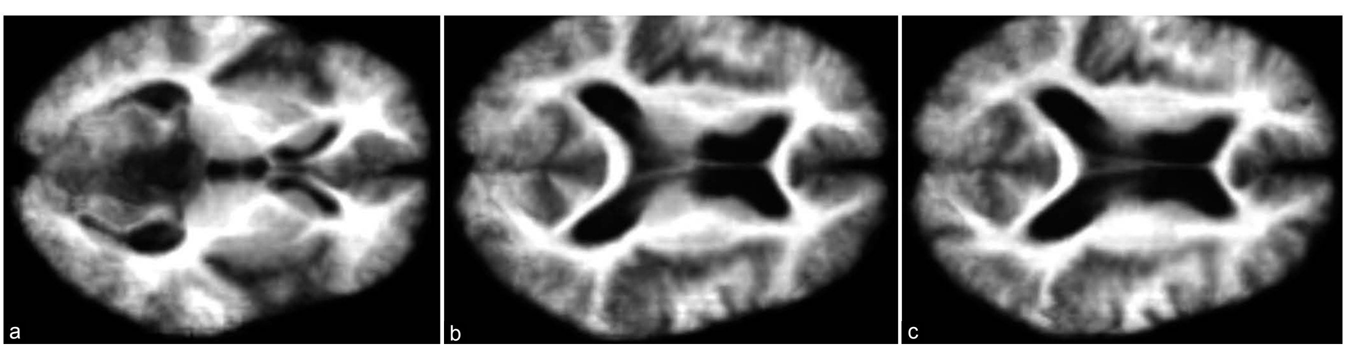

Figure 1: (a) Template image, (b) Reference image, and (c) Matched image.

Elastic registration, fluid registration, diffusion registration, curvature registration, and demons registration are some of the most well-known image registration algorithms and detailed information about each of these methods can be seen at [7] and at the references of it.

The first author developed [1] a 3–D image registration method in his Ph.D. thesis. The method was developed by adjusting the divergence and curl of an intermediate vector field appearing in the grid deformation method from which the deformation field is computed. The method incorporates the sum of squared differences as the similarity metric with a Lagrange multiplier optimization technique for the non-rigid image registration of three-dimensional images. The Poisson equations appearing in the coupled system resulting from the optimality system are solved, in a decoupled manner, by the finite element (FEM) and multigrid methods (MG), separately. Experimental results prove that this method is a quite efficient method as well. In [4], the first author compared this method with the methods mentioned in the previous paragraph.

An image registration method might be described as efficient if the quality of the registered images is good, duration of the registration process is short and the amount of the similarity measure is small. The quality of the matched images using aforementioned techniques is almost the same and the information about duration of registration and the amount of the similarity measure is given by 1.

Table 1: The similarity measure C_{sim} and the cost in computational time

Iterations	Present method		The method at [1]	
	C_{sim}	Cost	C_{sim}	Cost
1	1400	1 sec	2400	2 sec
2	1350	1.1 sec	2100	2 sec
15	880	2 sec	1200	4 sec
30	440	7 sec	500	16 sec
80	220	15 sec	200	34 sec
160	100	20 sec	100	55 sec
200	44	38 sec	22	1 mins

Table 2: The similarity measure C_{sim} and the cost in computational time

Iterations	AIMCM		PM	
	C_{sim}	cost	C_{sim}	cost
1	1674.6	1 sec	1674.6	2 sec
2	908.5	1.1 sec	1102.5	2 sec
15	160.7	2 sec	209.9	4 sec
30	40.7	7 sec	63.5	16 sec
80	15.8	15 sec	26.5	34 sec
160	10.7	20 sec	16.5	55 sec
240	6.4	38 sec	12.5	2 mins
400	3.9	55 sec	7.5	3 mins

As the Tables 1 and 2 and the Figures at 1 indicate the present method. which utilizes the advantages of both variational calculus and wavelets is a highly efficient medical image matching method.

6. Conclusion

In this paper, we introduce a hybrid method for deformable matching of MR images by exploiting the Gabor wavelet energy maps of MR images and techniques of variational calculus. Image matching problem is expressed as an optimal control problem and discretization of the resulting Euler-Lagrange equations is written in terms of the system of linear equations in the form of $Au=f$, where u is the image displacement field. The proposed algorithm succeeded in providing efficient matching of MR images. Experimental results proved that the method can match MR images better than the only variational or only wavelet-based methods. In a future work, we will investigate the applications of this image matching technique to the registration of noisy and blurred images.

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AUTHORS

Mehmet Ali Akinlar is an assistant professor of mathematics at Bilecik Seyh Edebali University. He completed his M.Sc. at McMaster University and Ph.D. at UT, Arlington in 2005 and 2009, respectively. Upon completion of his Ph.D, he worked as a post-doctoral research associate at department of scientific computing at Florida State University under the supervision of Max Gunzburger. His research interests lie at Numerical solution of differential equations and PDE-based image processing.

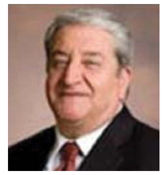
E-mail: mehmetaliakinlar@gmail.com

Muhammet Kurulay is an assistant professor of mathematics at Yildiz Technical University, Turkey. He obtained his Ph.D. from Yildiz Technical University, Turkey. He is an active researcher with broad research and has teaching experience in various universities of the world, including university of Connecticut, USA. He is a leading expert in fractional differential equations and has significant research studies on numerical analysis and applied mathematics.

E-mail: mkurulay@yildiz.edu.tr

Aydin Secer is an assistant professor of mathematics at department of mathematical engineering at Yildiz Technical University, Istanbul, Turkey. He is in the organizing committee of the International Conference: ICAAA2012. He is interested in scientific computing, computer programming, numerical methods as well as differential and integral equations.

E-mail: asecer@yildiz.edu.tr



Mehmet Celenk received his B.Sc. and M.Sc. degrees from Istanbul Technical University, in 1974 and 1976, respectively, both in electrical and communications engineering, and his Ph.D. degree from Stevens Institute of Technology in electrical engineering and computer science, in 1983. He joined Ohio University in 1985, where he is currently a Professor of Electrical Engineering and Computer Science. His research interests include image and video processing, computer vision, multimedia, data fusion, pattern recognition, and biomedical engineering. He has published over 200 articles in these fields. He supervised two visiting scholars, and 31 M.S./Ph.D. theses. Dr. Celenk was the recipient of the 1988 Fritz and Dolores Russ Research Award of the College of Engineering and Technology of Ohio University. He was awarded the Avionics Academic Challenge Faculty Fellowship at Ohio University from 1988 to 1992. He has been an Associate Editor of the IEEE Transactions on Systems, Man, and Cybernetics - Part A: Systems and Humans since 2005, a member of the Editorial Board of the Journal of Recent Patents on Signal Processing, since 2008.

E-mail: celenk@ohio.edu

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