mathematics

Article

Monad Metrizable Space

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Received: 18 September 2020; Accepted: 20 October 2020; Published: 31 October 2020

Abstract: Do the topologies of each dimension have to be same and metrizable for metricization of any space? I show that this is not necessary with monad metrizable spaces. For example, a monad metrizable space may have got any indiscrete topologies, discrete topologies, different metric spaces, or any topological spaces in each different dimension. I compute the distance in real space between such topologies. First, the passing points between different topologies is defined and then a monad metric is defined. Then I provide definitions and some properties about monad metrizable spaces and PAS metric spaces. I show that any PAS metric space is also a monad metrizable space. Moreover, some properties and some examples about them are presented.

Keywords: soft set theory; soft metric space; amply soft set; amply soft monad point; AS topology; PAS topology; PAS metric space; monad metrizable space; monad metric space; P_i, i = 0, 1, 2, 3, 4; parametric separation axioms

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1. Introduction

B. Greene [1] mentioned in his book "The Elegance of the Universe" that the 11th dimension appears when the Heterotic-E matching constant is greater than 1 but not less than 1. In this case, it may be thought that the 11th dimension is a different topology. I think this may be a discrete topology. So, is a topology possible in which different topologies of different dimensions can exist? For topologies of different sizes to be measured, should each topology be the same and measurable? In this context, for a topological space in which different topologies can be written in different parameters, it should be looked at whether a set is created in which different sets can be written in different parameters. So, let us first analyze Molodtsov's soft set [2], which makes it possible to write different sets with different parameters.

Molodtsov [2] defined a soft set and gave some properties about it. For this, he thought that there are many uncertainties to solve complicated problems such as in sociology, economics, engineering, medical science, environment problems, statistics, etc. There is no deal to solve them successfully. However, there are some theories such as vague sets theory [3], fuzzy sets [4], probability, intuitionistic fuzzy sets [5], rough sets [6], interval mathematics [7], etc., but these studies have their own complexities.

Maji et al. [8] defined soft subset, soft superset, soft equality and they gave some operators such as intersection, union of two soft sets, and complement of a soft set. They presented some properties about them. But some properties of them are false. So, Yang [9] gave counterexample about some of them. Also, Ali et al. [10] gave counter example about the others. Then they redefined operations of soft sets. Then the other researchers pointed out these false about soft operators and they gave some new definitions about soft operators of soft sets [11–19]. These are so valuable studies.

Up to now, there are many studies on the soft sets and their operations, also, taking universal set of parameter E is finite and countable because of the definition of the soft set. There are many soft sets and their operators defined. As first, in the soft set defined by Molodtsov [2] and developed by Maji et al. [8] the union of any soft set and its complement need not be a universal set. So, this situation makes a lot of deficiencies like a problem about complement of any soft set. To overcome this problem, researchers see two different ways. One way is using fixed parameter set in soft sets like Shabir and Naz's work [20]. They defined a lot of concept about a soft set and its topology. They used same parameter sets in their soft sets. This situation limits the soft topology. As a result of the fixed parameter, a soft point could be defined with fixed parameter. As a result of this, all soft sets have same values in all their parameters [21–24]. Some researchers tried to overcome this situation by defining another soft point [11,18,21,25]. You can see the latest study examples based on Shabir and Naz's work [26–32]. The other way is to redefine soft set and its operations. As first, Çağman and Enginoğlu [13] redefined soft set and its operations. This study is so valuable. But anyone could not take uncountable or infinite universal parameter set in practice. Because it is not pointed out what will we do with parameter sets between two soft sets exactly while using soft operators. You can see the latest study examples based on Çağman and Enginoglu's work [26,30–34]. Çağman et al. [35] defined a soft topology. They use different parameter sets in their soft sets. But all of them are finite because of the definition of soft operations. Then, Zhu and Wen [19] redefined a soft set and gave operations of it. Also, they pointed out what will we do with parameters set between two soft sets while using soft operators. But it is so complicated and as a result, cause same problems. You can see the latest study examples based on Zhu and Wen's work [36–38]. A similar situation drew attention of Fatimah et al. in [39]. They said that in their study all soft and hybrid soft sets used so far binary operation (either 0 or 1) or else real numbers between 0 and 1. So they defined a new soft set and it is called an N-Soft set. They used n parameter in their study, $n \in \mathbb{N}$, is natural numbers. Riaz et al. defined an N-Soft topology in [30]. Therefore, we can say that they use set of parameters E as infinite and countable. They use initial universe X as finite and countable. But in fact, in real life or in space they do not have to be finite and countable.

In order to overcome all the problems mentioned about above Göçür O [40] defined an amply soft set. He named this soft set as an amply soft set, together with its operations, in order to eliminate the complexity by selecting the ones that are suitable for a certain purpose among the previously defined soft sets and the operations between them and redefining otherwise. Amply soft sets use any kind of universal parameter set and initial universe (such as finite or infinite, countable or uncountable). Also, he introduced subset, superset, equality, empty set, whole set about amply soft sets. And he gave operations such as union, intersection, difference of two amply soft sets and complement of an amply soft set. Then he defined three different amply soft points such as amply soft whole point, amply soft point, monad point. He also gave examples related taking universal set as uncountable.

Göçür O. defined a new soft topology, and it is called as a PAS topology in [40]. The PAS topology allows to write different elements of classical topologies in its each parameter sets. The classical topologies may be finite, infinite, countable or uncountable. This situation removes all of boundaries in a soft topology and cause it to spread over larger areas. A PAS topology is a special case of an AS topology. For this purpose, he defined a new soft topology, and it is called as an amply soft topology or briefly an AS topology. He gave parametric separation axioms which are different from T_i separation axioms. T_i questions the relationship between the elements of space itself while P_i questions the strength of the connection between their parameters.

The main aim of this study is to define monad metrizable spaces. A monad metrizable space may have got any topological spaces and any different metric spaces in each different dimension. The distance in real space between these topologies is computed. First of all, I define passing points between different topologies, and define a monad metric. Then I give a monad metrizable space and a PAS metric space. I show that any PAS metric space is also a monad metrizable space. Some properties and some examples about them are also presented.

2. Materials and Methods

These terminologies are used hereafter in the paper: X denotes an initial universe, E denotes a universal set of parameters; $A,\ B,\ C$ are subsets of E.

Definition 1. *Ref [40]. Let $P(X)$ denote the power set of X. If $F : E \to P(X)$ is a mapping given by*

$$F(e) = \begin{cases} F(e), & \forall e \in A; \\ \varnothing, & \forall e \in E - A, \end{cases} \tag{1}$$

then F with A is called as an amply soft set over X and it is denoted by $F * A$. We can say an AS set instead of an amply soft set for briefness.

Example 1. *Ref [40]. Let $X = \{x_1, x_2, x_3, x_4\}$ be a universal set, $E = \{e_1, e_2, e_3, e_4, e_5\}$ a set of parameters and $A = \{e_1, e_2, e_4, e_5\}$ a subset of E. Let $F : A \to P(X)$ be the mapping given by $F(e_1) = \{x_1, x_2\}$, $F(e_2) = \{x_2, x_4\}$, $F(e_4) = \{x_3\}$, $F(e_5) = \{x_3\}$. Then we can show it looks like the following:*

$$F * A = \left\{ \{x_1\}_{\{e_1\}}, \{x_2\}_{\{e_1,e_2\}}, \{x_3\}_{\{e_4,e_5\}}, \{x_4\}_{\{e_2\}} \right\}. \tag{2}$$

Definition 2. *Ref [40]. Let $F * A$ and $G * B$ be two amply soft sets over X. $F * A$ is subset of $G * B$, denoted by $F * A \widetilde{\subseteq} G * B$, if $F(e) \subseteq G(e)$, for all $e \in A$.*

Definition 3. *Ref [40]. Let $F * A$ and $G * B$ be two amply soft sets over X. $F * A$ is superset of $G * B$, denoted by $F * A \widetilde{\supseteq} G * B$, if $F(e) \supseteq G(e)$, for all $e \in B$.*

Definition 4. *Ref [40]. Let $F * A$ and $G * B$ be two amply soft sets over X. If $F * A$ is subset of $G * B$ and $G * B$ is subset of $F * A$ also, then $F * A$ and $G * B$ are said to be equal and denoted by $F * A \widetilde{=} G * B$.*

Definition 5. *Ref [40]. An amply soft set $F * A$ over X is said to be an empty amply soft set denoted by $\widetilde{\varnothing}$ if $F(e) = \varnothing$ for all $e \in E$.*

Definition 6. *Ref [40]. An amply soft set $F * E$ over X is said to be an absolute amply soft set denoted by $\widetilde{X}$ if for all $e \in E$, $F(e) = X$.*

Definition 7. *Ref [40]. The union of two amply soft sets of $F * A$ and $G * B$ over a common universe X is the amply soft set $H * C$, where $C = A \cup B$ and for all $e \in E$,*

$$H(e) = \begin{cases} \mathrm{F(e)},\ \forall e \in A - B, \\ \mathrm{G(e)},\ \forall e \in B - A, \\ \mathrm{F(e)} \cup \mathrm{G(e)},\ \forall e \in A \cap B, \\ \varnothing,\ \forall e \in E - C. \end{cases} \tag{3}$$

We can write $F * A \widetilde{\cup} G * B \widetilde{=} F \cup G * A \cup B \widetilde{=} H * C$.

Definition 8. *Ref [40]. The intersection $H * C$ of two amply soft sets $F * A$ and $G * B$ over a common universe X denoted by $F * A \widetilde{\cap} G * B$ is defined as $C = A \cap B$ and for all $e \in E$,*

$$H(e) = \begin{cases} \mathrm{F(e)} \cap \mathrm{G(e)}, \forall e \in C, \\ \varnothing, \forall e \in E - C. \end{cases} \tag{4}$$

We can write $F * A \widetilde{\cap} G * B \widetilde{=} F \cap G * A \cap B \widetilde{=} H * C$.

Definition 9. *Ref [40]. The difference $H*A$ of two amply soft sets $F * A$ and $G * B$ over X is denoted by $F * A \widetilde{\backslash} G * B$ and it is defined as*

$$H(e) = \begin{cases} \mathrm{F(e)}, \forall e \in A - B, \\ \mathrm{F(e)} \backslash \mathrm{G(e)}, \forall e \in A \cap B, \\ \varnothing, \forall e \in E - A. \end{cases} \tag{5}$$

We can write $F * A \widetilde{\backslash} G * B \cong F \backslash G * A \cong H * A$.

Definition 10. *Ref [40]. Let* $F * A$ *be an amply soft set over* $\widetilde{X}$. *The complement of an amply soft set* $F * A$ *over* X *is denoted by* $(F * A)^{\widetilde{\prime}} \cong F' * E$ *where* $F' : E \to P(X)$ *is a mapping defined as* $F'(e) = X - F(e)$ *for all* $e \in E$.

Note that, $F' : E \to P(X)$ *is a mapping given by*

$$F'(e) = \begin{cases} X - F(e), \ \forall e \in A, \\ X, \ \forall e \in E - A. \end{cases} \tag{6}$$

Example 2. *Ref [40]. Let* $X = \{h_1, h_2, h_3, h_4, h_5\}$, $E = \{e_1, e_2, e_3, e_4, e_5\}$ *and its subsets* $A = \{e_1, e_2, e_3\}$, $B = \{e_1, e_3, e_4\}, C = \{e_1, e_4\}$ *and* $F * A, G * B, H * C$ *are amply soft sets over* X *defined as follows respectively,*

$$F * A \cong \left\{ \{h_1, h_2\}_{\{e_1\}}, \ \{h_2, h_3, h_4\}_{\{e_2\}}, \ \{h_1, h_2, h_5\}_{\{e_3\}} \right\}, \tag{7}$$

$$G * B \cong \left\{ \{h_1, h_4\}_{\{e_1\}}, \{h_1, h_2, h_4\}_{\{e_3\}}, \{h_3, h_5\}_{\{e_4\}} \right\}, \tag{8}$$

$$H * C \cong \left\{ \{h_1, h_2\}_{\{e_1\}}, \ \{h_1, h_2, h_3\}_{\{e_4\}} \right\}. \tag{9}$$

Then,

$$F * A \widetilde{\cup} G * B \cong F \cup G * A \cup B \cong \left\{ \{h_1, h_2, h_4\}_{\{e_1\}}, \ \{h_2, h_3, h_4\}_{\{e_2\}}, \ \{h_1, h_2, h_4, h_5\}_{\{e_3\}}, \ \{h_3, h_5\}_{\{e_4\}} \right\}. \tag{10}$$

$$F * A \widetilde{\cap} G * B \cong F \cap G * A \cap B \cong \left\{ \{h_1\}_{\{e_1\}}, \ \{h_1, h_2\}_{\{e_3\}} \right\}. \tag{11}$$

$$F * A \widetilde{\backslash} G * B \cong F \backslash G * A \cong \left\{ \{h_2\}_{\{e_1\}}, \ \{h_2, h_3, h_4\}_{\{e_2\}}, \{h_5\}_{\{e_3\}} \right\}. \tag{12}$$

$$G * B \widetilde{\backslash} F * A \cong G \backslash F * B \cong \left\{ \{h_4\}_{\{e_1, e_3\}}, \ \{h_3, h_5\}_{\{e_4\}} \right\}. \tag{13}$$

$$(F * A)^{\widetilde{\prime}} \cong F' * E \cong \left\{ \{h_3, h_4, h_5\}_{\{e_1\}}, \ \{h_1, h_5\}_{\{e_2\}}, \ \{h_3, h_4\}_{\{e_3\}}, \ X_{\{e_4, e_5\}} \right\}. \tag{14}$$

$$(G * B)^{\widetilde{\prime}} \cong G' * E \cong \left\{ \{h_2, h_3, h_5\}_{\{e_1\}}, \ \{h_3, h_5\}_{\{e_3\}}, \ \{h_1, h_2, h_4\}_{\{e_4\}}, \ X_{\{e_2, e_5\}} \right\}. \tag{15}$$

Proposition 1. *Ref [40]. Let* $F * A$, $G * B$ *and* $H * C$ *be amply soft sets over* X; $A, B, C \subseteq E$. *Then the following holds.*

$$F * A \widetilde{\cup} F * A \cong F * A, \tag{16}$$

$$F * A \widetilde{\cap} F * A \cong F * A, \tag{17}$$

$$F * A \widetilde{\cup} \widetilde{\varnothing} \cong F * A, \tag{18}$$

$$F * A \widetilde{\cap} \widetilde{\varnothing} \cong \widetilde{\varnothing}, \tag{19}$$

$$F * A \widetilde{\cup} \widetilde{X} \cong \widetilde{X}, \tag{20}$$

$$F * A \widetilde{\cap} \widetilde{X} \cong F * A, \tag{21}$$

$$F * A \widetilde{\cup} G * B \cong G * B \widetilde{\cup} F * A, \tag{22}$$

$$F * A \widetilde{\cap} G * B \cong G * B \widetilde{\cap} F * A, \tag{23}$$

$$\left(F * A \widetilde{\cup} G * B \right) \widetilde{\cup} H * C \cong F * A \widetilde{\cup} \left(G * B \widetilde{\cup} H * C \right), \tag{24}$$

$$\left(F * A \widetilde{\cap} G * B \right) \widetilde{\cap} H * C \cong F * A \widetilde{\cap} \left(G * B \widetilde{\cap} H * C \right), \tag{25}$$

$$F * A \widetilde{\cap} \left(G * B \widetilde{\cup} H * C \right) \cong \left(F * A \widetilde{\cap} G * B \right) \widetilde{\cup} \left(F * A \widetilde{\cap} H * C \right), \tag{26}$$

$$F*A\widetilde{\cup}\left(G*B\widetilde{\cap}H*C\right)\widetilde{=}\left(F*A\widetilde{\cup}G*B\right)\widetilde{\cap}\left(F*A\widetilde{\cup}H*C\right). \quad (27)$$

$$\left((F*A)^{\widetilde{\prime}}\right)^{\widetilde{\prime}}\widetilde{=}F*A \quad (28)$$

$$\left(F*A\widetilde{\cup}G*B\right)^{\widetilde{\prime}}\widetilde{=}(F*A)^{\widetilde{\prime}}\widetilde{\cap}(G*B)^{\widetilde{\prime}} \quad (29)$$

$$\left(F*A\widetilde{\cap}G*B\right)^{\widetilde{\prime}}\widetilde{=}(F*A)^{\widetilde{\prime}}\widetilde{\cup}(G*B)^{\widetilde{\prime}} \quad (30)$$

Definition 11. *Ref [40]. Let* $\{a\}\subset E$ *and let* $F*\{a\}$ *be an amply soft set over* $X,\ x\in X$. *If* $F*\{a\}$ *is defined as* $F(a)=\{x\}$, *then* $F*\{a\}$ *is called as a monad point and it is denoted by* x_a.

Definition 12. *Ref [40]. Let* $a\in A$ *and* $F*A$ *be an amply soft set over* $X,\ x\in X$. *We say that* $x_a\widetilde{\in}F*A$ *read as monad point* x *belongs to the amply soft set* $F*A$ *if* $x\in F(a)$.

Definition 13. *Ref [40]. Let* $a\in E$ *and* $F*\{a\}$ *be an amply soft set over* $X,\ x\in X$. *We say that* $x_a\widetilde{\notin}F*\{a\}$ *if* $x\notin F(a)$.

Definition 14. *Ref [40]. Let* $\widetilde{\tau}$ *be the collection of amply soft sets over* X, *then* $\widetilde{\tau}$ *is said to be an amply soft topology (or briefly AS topology) on* X *if,*

1. $\varnothing, X$ *belong to* $\widetilde{\tau}$
2. *The union of any number of amply soft sets in* $\widetilde{\tau}$ *belongs to* $\widetilde{\tau}$
3. *The intersection of any two amply soft sets in* $\widetilde{\tau}$ *belongs to* $\widetilde{\tau}$.

The triplet $\left(\widetilde{X},\widetilde{\tau},E\right)$ *is called as an amply soft topological space over* $\widetilde{X}$.
We will use AS topological space $\widetilde{X}$ *instead of amply soft topological space* $\left(\widetilde{X},\widetilde{\tau},E\right)$ *for shortly.*

Definition 15. *Ref [40]. Let* $\left(\widetilde{X},\widetilde{\tau},E\right)$ *be an AS topological space, then the members of* $\widetilde{\tau}$ *are said to be AS open sets in an AS topological space* $\widetilde{X}$.

Definition 16. *Ref [40]. Let* $\left(\widetilde{X},\widetilde{\tau},E\right)$ *be an AS topological space. An amply soft set* $F*A$ *over* $\widetilde{X}$ *is said to be an AS closed set in an AS topological space* $\widetilde{X}$, *if its complement* $(F*A)^{\widetilde{\prime}}$ *belongs to* $\widetilde{\tau}$.

Proposition 2. *Ref [40].*

1. *Let* $\left(\widetilde{X},\widetilde{\tau},E\right)$ *be an AS topological space. Then*
2. $\widetilde{\varnothing},\widetilde{X}$ *are AS closed sets in AS topological space* $\widetilde{X}$,
3. *The intersection of any number of AS closed sets is an AS closed set in an AS topological space* $\widetilde{X}$,

The union of any two AS closed sets is an AS closed set in an AS topological space $\widetilde{X}$.

Proposition 3. *Ref [40]. Let* $\left(\widetilde{X},\widetilde{\tau},E\right)$ *be an AS topological space. Then the collection* $\tau_e=\left\{F(e)\middle|F*E\widetilde{\in}\widetilde{\tau}\right\}$ *for each* $e\in E$, *defines topologies on* X.

Definition 17. *Ref [40]. Let* $\left(\widetilde{X},\widetilde{\tau},E\right)$ *be an AS topological space and* $\widetilde{\beta}\widetilde{\subseteq}\widetilde{\tau}$. *If every element of* $\widetilde{\tau}$ *can be written as any union of elements of* $\widetilde{\beta}$, *then* $\widetilde{\beta}$ *is called as an AS basis for the AS topology* $\widetilde{\tau}$. *Then we can say that each element of* $\widetilde{\beta}$ *is an AS basis element.*

Definition 18. *Ref [40]. Let* $n\in\mathbb{N},\ e_n\in E$. *Let* (X,τ_n) *be topological spaces over same initial universe* X. *Let* f *be mapping from* τ_n *to* τ_{e_n} *for all* $e\in E$. *Then* $\beta=\{\tau_{e_n}\}$ *is an AS basis for an AS topology* $\widetilde{\tau}$. *We can say that it is called as an AS topology produced by classical topology and for shortly a PAS topology.*

Note that these separation axioms defined as parametric separations in below are different from T_i separation axioms. T_i questions the relationship between the elements of space itself while P_i questions the strength of the connection between their parameters.

Definition 19. *Ref [40]. Let* $\left(\widetilde{X},\widetilde{\tau},E\right)$ *be an AS topological space and* $a,b \in E$ *such that* $a \neq b$*, if there exist* $x \in X$ *and AS open sets* $F*A$ *and* $G*B$ *such that* $x_a \widetilde{\in} F*A$ *and* $x_b \widetilde{\notin} F*A$*; or* $x_b \widetilde{\in} G*B$ *and* $x_a \widetilde{\notin} G*B$*, then* $\left(\widetilde{X},\widetilde{\tau},E\right)$ *is called as a* P_0 *space.*

Definition 20. *Ref [40]. Let* $\left(\widetilde{X},\widetilde{\tau},E\right)$ *be an AS topological space and* $a,b \in E$ *such that* $a \neq b$*. If there exist* $x \in X$ *and AS open sets* $F*A$ *and* $G*B$ *such that* $x_a \widetilde{\in} F*A$ *and* $x_b \widetilde{\notin} F*A$*; and* $x_b \widetilde{\in} G*B$ *and* $x_a \widetilde{\notin} G*B$*, then* $\left(\widetilde{X},\widetilde{\tau},E\right)$ *is called as a* P_1 *space.*

Theorem 1. *Ref [40]. Let* $\left(\widetilde{X},\widetilde{\tau},E\right)$ *be a* P_1 *space, then it is also a* P_0 *space.*

Definition 21. *Ref [40]. Let* $\left(\widetilde{X},\widetilde{\tau},E\right)$ *be an AS topological space and* $a,b \in E$ *such that* $a \neq b$*. If there exist* $x \in X$ *and AS open sets* $F*A$ *and* $G*B$ *such that* $x_a \widetilde{\in} F*A$*,* $x_b \widetilde{\in} G*B$ *and* $F*A \widetilde{\cap} G*B \widetilde{=} \widetilde{\varnothing}$*, then* $\left(\widetilde{X},\widetilde{\tau},E\right)$ *is called as a* P_2 *space.*

Theorem 2. *Ref [40]. Let* $\left(\widetilde{X},\widetilde{\tau},E\right)$ *be a* P_2 *space, then it is also a* P_1 *space.*

Theorem 3. *Ref [40]. Any PAS topological space* $\left(\widetilde{X},\widetilde{\tau},E\right)$ *is a* P_2 *space.*

Example 3. *Ref [40]. Let* $e_1, e_2 \in E$*,* $x \in X$ *and* (X,τ_{e_1}) *and* (X,τ_{e_2}) *be indiscrete topological spaces over same universe* X*. Let* $\left(\widetilde{X},\widetilde{\tau},E\right)$ *be their PAS topology. So;* $\widetilde{\tau} = \left\{\widetilde{\varnothing},\ \widetilde{X},\ \left\{X_{\{e_1\}}\right\},\ \left\{X_{\{e_2\}}\right\}\right\}$ *is an AS topological space over* $\widetilde{X}$*.*

Therefore there exist AS open sets $\left\{X_{\{e_1\}}\right\}$ *and* $\left\{X_{\{e_2\}}\right\}$ *such that* $x_{e_1} \widetilde{\in} \left\{X_{\{e_1\}}\right\}$*,* $x_{e_2} \widetilde{\in} \left\{X_{\{e_2\}}\right\}$ *and* $\left\{X_{\{e_1\}}\right\} \widetilde{\cap} \left\{X_{\{e_2\}}\right\} \widetilde{=} \widetilde{\varnothing}$*. So* $\left(\widetilde{X},\widetilde{\tau},E\right)$ *is a* P_2 *space.*

Definition 22. *Ref [40]. Let* $\left(\widetilde{X},\widetilde{\tau},E\right)$ *be an AS topological space,* $H*C$ *be an AS closed set,* $a \in E$*,* $x \in X$ *such that* $x_a \widetilde{\notin} H*C$*. If there exist* $x \in X$ *and AS open sets* $F*A$ *and* $G*B$ *such that* $x_a \widetilde{\in} F*A$*,* $H*C \widetilde{\subseteq} G*B$ *and* $F*A \widetilde{\cap} G*B \widetilde{=} \widetilde{\varnothing}$ *then* $\left(\widetilde{X},\widetilde{\tau},E\right)$ *is called as a Halime space.*

Definition 23. *Ref [40]. Let* $\left(\widetilde{X},\widetilde{\tau},E\right)$ *be an AS topological space. Then it is said to be a* P_3 *space if it is both a Halime space and a* P_1 *space.*

Example 4. *Ref [40]. Let* $X = \{x,y,z\}$ *be a universal set,* $E = \{e_1,e_2,e_3\}$ *be a parameter set and let* $\widetilde{\tau} = \left\{\widetilde{\varnothing},\ \widetilde{X},\ \left\{\{x,y\}_{\{e_1\}}, \{x,z\}_{\{e_2\}}, \{y,z\}_{\{e_3\}}\right\},\ \left\{\{z\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{x\}_{\{e_3\}}\right\}\right\}$ *such that every members be amply soft set* $E \to P(X)$*. Then* $\left(\widetilde{X},\widetilde{\tau},E\right)$ *is an AS topological space. Let us see if it is a* P_3 *space.*

Firstly, let us see if it is a Halime space.

For $e_1 \in E$*, choose* x_{e_1} *and for an AS closed set* $\left\{\{z\}_{\{e_1\}},\ \{y\}_{\{e_2\}},\ \{x\}_{\{e_3\}}\right\}$ *such that* $x_{e_1} \widetilde{\notin} \left\{\{z\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{x\}_{\{e_3\}}\right\}$*. There exists an AS open set* $\left\{\{z\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{x\}_{\{e_3\}}\right\}$ *such that* $\left\{\{z\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{x\}_{\{e_3\}}\right\} \widetilde{\subseteq} \left\{\{z\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{x\}_{\{e_3\}}\right\}$*. There exists an AS open set* $\left\{\{x,y\}_{\{e_1\}}, \{x,z\}_{\{e_2\}}, \{y,z\}_{\{e_3\}}\right\}$ *such that* $x_{e_1} \widetilde{\in} \left\{\{x,y\}_{\{e_1\}}, \{x,z\}_{\{e_2\}}, \{y,z\}_{\{e_3\}}\right\}$ *and* $\left\{\{x,y\}_{\{e_1\}}, \{x,z\}_{\{e_2\}}, \{y,z\}_{\{e_3\}}\right\} \widetilde{\cap} \left\{\{z\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{x\}_{\{e_3\}}\right\} \widetilde{=} \widetilde{\varnothing}$*,*

For $e_2 \in E$*, choose* x_{e_2} *and for an AS closed set* $\left\{\{z\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{x\}_{\{e_3\}}\right\}$ *such that* $x_{e_2} \widetilde{\notin} \left\{\{z\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{x\}_{\{e_3\}}\right\}$*. There exists an AS open set* $\left\{\{z\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{x\}_{\{e_3\}}\right\}$ *such that* $\left\{\{z\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{x\}_{\{e_3\}}\right\} \widetilde{\subseteq} \left\{\{z\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{x\}_{\{e_3\}}\right\}$*. There exists an AS open set* $\left\{\{x,y\}_{\{e_1\}}, \{x,z\}_{\{e_2\}}, \{y,z\}_{\{e_3\}}\right\}$ *such that* $x_{e_2} \widetilde{\in} \left\{\{x,y\}_{\{e_1\}}, \{x,z\}_{\{e_2\}}, \{y,z\}_{\{e_3\}}\right\}$ *and* $\left\{\{x,y\}_{\{e_1\}}, \{x,z\}_{\{e_2\}}, \{y,z\}_{\{e_3\}}\right\} \widetilde{\cap} \left\{\{z\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{x\}_{\{e_3\}}\right\} \widetilde{=} \widetilde{\varnothing}$*, For* $e_3 \in E$*, choose* z_{e_3} *and for an AS closed set*

$\left\{\{z\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{x\}_{\{e_3\}}\right\}$ *such that* $z_{e_3} \widetilde{\notin} \left\{\{z\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{x\}_{\{e_3\}}\right\}$ *There exists an AS open set* $\left\{\{z\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{x\}_{\{e_3\}}\right\}$ *such that* $\left\{\{z\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{x\}_{\{e_3\}}\right\} \widetilde{\subseteq} \left\{\{z\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{x\}_{\{e_3\}}\right\}$. *There exists an AS open set* $\left\{\{x,y\}_{\{e_1\}}, \{x,z\}_{\{e_2\}}, \{y,z\}_{\{e_3\}}\right\}$ *such that* $z_{e_3} \widetilde{\in} \left\{\{x,y\}_{\{e_1\}}, \{x,z\}_{\{e_2\}}, \{y,z\}_{\{e_3\}}\right\}$ *and* $\left\{\{x,y\}_{\{e_1\}}, \{x,z\}_{\{e_2\}}, \{y,z\}_{\{e_3\}}\right\} \widetilde{\cap} \left\{\{z\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{x\}_{\{e_3\}}\right\} \cong \widetilde{\varnothing}$,

So $\left(\widetilde{X}, \widetilde{\tau}, E\right)$ is a Halime space from the Definition 22.

Finally, let us see if it is a P_1 space.

For $e_1, e_2 \in E$, *there exist* $y \in X$ *and AS open sets* $\left\{\{x,y\}_{\{e_1\}}, \{x,z\}_{\{e_2\}}, \{y,z\}_{\{e_3\}}\right\}$ *and* $\left\{\{z\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{x\}_{\{e_3\}}\right\}$ *such that* $y_{e_1} \widetilde{\in} \left\{\{x,y\}_{\{e_1\}}, \{x,z\}_{\{e_2\}}, \{y,z\}_{\{e_3\}}\right\}$ *and* $y_{e_2} \widetilde{\notin} \left\{\{x,y\}_{\{e_1\}}, \{x,z\}_{\{e_2\}}, \{y,z\}_{\{e_3\}}\right\}$; *and* $y_{e_2} \in \left\{\{z\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{x\}_{\{e_3\}}\right\}$ *and* $y_{e_1} \widetilde{\notin} \left\{\{z\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{x\}_{\{e_3\}}\right\}$.

For $e_1, e_3 \in E$, *there exist* $x \in X$ *and AS open sets* $\left\{\{x,y\}_{\{e_1\}}, \{x,z\}_{\{e_2\}}, \{y,z\}_{\{e_3\}}\right\}$ *and* $\left\{\{z\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{x\}_{\{e_3\}}\right\}$ *such that* $x_{e_1} \widetilde{\in} \left\{\{x,y\}_{\{e_1\}}, \{x,z\}_{\{e_2\}}, \{y,z\}_{\{e_3\}}\right\}$ *and* $x_{e_3} \widetilde{\notin} \left\{\{x,y\}_{\{e_1\}}, \{x,z\}_{\{e_2\}}, \{y,z\}_{\{e_3\}}\right\}$; *and* $x_{e_3} \widetilde{\in} \left\{\{z\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{x\}_{\{e_3\}}\right\}$ *and* $x_{e_1} \widetilde{\notin} \left\{\{z\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{x\}_{\{e_3\}}\right\}$.

For $e_2, e_3 \in E$, *there exist* $y \in X$ *and AS open sets* $\left\{\{x,y\}_{\{e_1\}}, \{x,z\}_{\{e_2\}}, \{y,z\}_{\{e_3\}}\right\}$ *and* $\left\{\{z\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{x\}_{\{e_3\}}\right\}$ *such that* $y_{e_3} \widetilde{\in} \left\{\{x,y\}_{\{e_1\}}, \{x,z\}_{\{e_2\}}, \{y,z\}_{\{e_3\}}\right\}$ *and* $y_{e_2} \widetilde{\notin} \left\{\{x,y\}_{\{e_1\}}, \{x,z\}_{\{e_2\}}, \{y,z\}_{\{e_3\}}\right\}$; *and* $y_{e_2} \widetilde{\in} \left\{\{z\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{x\}_{\{e_3\}}\right\}$ *and* $y_{e_3} \notin \left\{\{z\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{x\}_{\{e_3\}}\right\}$.

Therefore, $\left(\widetilde{X}, \widetilde{\tau}, E\right)$ is a P_1 space from the Definition 20. $\left(\widetilde{X}, \widetilde{\tau}, E\right)$ is a P_3 space because it is both a Halime space and a P_1 space.

Theorem 4. *Ref [40]. Let* $\left(\widetilde{X}, \widetilde{\tau}, E\right)$ *be a* P_3 *space, then it is also a* P_2 *space.*

Definition 24. *Ref [40]. Let* $\left(\widetilde{X}, \widetilde{\tau}, E\right)$ *be an AS topological space,* $H * C$ *and* $K * D$ *be AS closed sets such that* $H * C \widetilde{\cap} K * D = \widetilde{\varnothing}$. *If there exist AS open sets* $F * A$ *and* $G * B$ *such that* $H * C \widetilde{\subseteq} F * A$, $K * D \widetilde{\subseteq} G * B$ *and* $F * A \widetilde{\cap} G * B \widetilde{=} \widetilde{\varnothing}$ *then* $\left(\widetilde{X}, \widetilde{\tau}, E\right)$ *is called as an Orhan space.*

Definition 25. *Ref [40]. Let* $\left(\widetilde{X}, \widetilde{\tau}, E\right)$ *be an AS topological space. Then it is said to be a* P_4 *space if it is both an Orhan space and a* P_1 *space.*

Example 5. *Ref [40]. Let us consider the AS topology* $\left(\widetilde{X}, \widetilde{\tau}, E\right)$ *on the Example 4. Let us see if it is a* P_4 *space.*

$\left\{\{x,y\}_{\{e_1\}}, \{x,z\}_{\{e_2\}}, \{y,z\}_{\{e_3\}}\right\}$ and $\left\{\{z\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{x\}_{\{e_3\}}\right\}$ are disjoint AS closed sets on it. We know that these are also disjoint AS open sets. So $\left(\widetilde{X}, \widetilde{\tau}, E\right)$ is an Orhan Space. Also $\left(\widetilde{X}, \widetilde{\tau}, E\right)$ is a P_1 space from the Example 4, so $\left(\widetilde{X}, \widetilde{\tau}, E\right)$ is a P_4 space from the Definition 25.

Example 6. *Ref [40]. Let us consider the PAS topology on the Example 3. Let us see if it is a* P_4 *space.*

First, let us see if it is an Orhan space. There exist AS closed sets $\left\{X_{\{e_1\}}\right\}$ *and* $\left\{X_{\{e_2\}}\right\}$ *such that* $\left\{X_{\{e_1\}}\right\} \widetilde{\cap} \left\{X_{\{e_2\}}\right\} \cong \widetilde{\varnothing}$. *We know that* $\left\{X_{\{e_1\}}\right\}$ *and* $\left\{X_{\{e_2\}}\right\}$ *are also AS open sets such that* $\left\{X_{\{e_1\}}\right\} \widetilde{\subseteq} \left\{X_{\{e_1\}}\right\}$, $\left\{X_{\{e_2\}}\right\} \widetilde{\subseteq} \left\{X_{\{e_2\}}\right\}$ *and* $\left\{X_{\{e_1\}}\right\} \widetilde{\cap} \left\{X_{\{e_2\}}\right\} \cong \widetilde{\varnothing}$. *So* $\left(\widetilde{X}, \widetilde{\tau}, E\right)$ *is an Orhan space.*

Finally, let us see if it is a P_1 *space.*

There exist AS open sets $\left\{X_{\{e_1\}}\right\}$ *and* $\left\{X_{\{e_2\}}\right\}$ *such that* $x_{e_1} \widetilde{\in} \left\{X_{\{e_1\}}\right\}$, $x_{e_2} \widetilde{\in} \left\{X_{\{e_1\}}\right\}$ *and* $x_{e_2} \widetilde{\in} \left\{X_{\{e_2\}}\right\}$, $x_{e_1} \widetilde{\notin} \left\{X_{\{e_2\}}\right\}$. *So* $\left(\widetilde{X}, \widetilde{\tau}, E\right)$ *is a* P_1 *space. Because* $\left(\widetilde{X}, \widetilde{\tau}, E\right)$ *is both an Orhan space and a* P_1 *space, it is a* P_4 *space.*

Proposition 4. *Ref [40]. Any PAS topological space* $\left(\widetilde{X}, \widetilde{\tau}, E\right)$ *may not be a* P_4 *space.*

Example 7. *Ref [40]. Let* $\mathbb{R}$ *be real numbers,* $E = \{e_1, e_2\}$ *be a universal parameter set,* $(\mathbb{R}, \tau_1)$ *be a discrete topological space and* $(\mathbb{R}, \tau_2)$ *be a finite complement topological space and from the Definition 18, their PAS topology over* $\widetilde{\mathbb{R}}$ *be* $\left(\widetilde{\mathbb{R}}, \widetilde{\tau}, E\right)$.

Now let us see if $\widetilde{\mathbb{R}}$ is a P_4 space.

Firstly, we can say that the PAS topology is a P_1 space because of the Theorem 3. and the Theorem 2. Let us see it,

For e_1, e_2; *there exist* $2 \in \mathbb{R}$ *and* $\left\{\{2\}_{\{e_1\}}, (\mathbb{R} - \{2,3,5\})_{\{e_2\}}\right\}$, $\left\{\{3\}_{\{e_1\}}, (\mathbb{R} - \{3,5\})_{\{e_2\}}\right\} \widetilde{\in} \widetilde{\tau}$ *such that* $2_{e_1} \widetilde{\in} \left\{\{2\}_{\{e_1\}}, (\mathbb{R} - \{2,3,5\})_{\{e_2\}}\right\}$ *and* $2_{e_2} \widetilde{\notin} \left\{\{2\}_{\{e_1\}}, (\mathbb{R} - \{2,3,5\})_{\{e_2\}}\right\}$; $2_{e_2} \widetilde{\in} \left\{\{3\}_{\{e_1\}}, (\mathbb{R} - \{3,5\})_{\{e_2\}}\right\}$ *and* $2_{e_1} \widetilde{\notin} \left\{\{3\}_{\{e_1\}}, (\mathbb{R} - \{3,5\})_{\{e_2\}}\right\}$. *Therefore, it is clearly seen that* $\left(\widetilde{\mathbb{R}}, \widetilde{\tau}, E\right)$ *is a* P_1 *space.*

Finally, let us see if $\widetilde{\mathbb{R}}$ *is an Orhan space. Let* $U, V \widetilde{\subseteq} \mathbb{R}$ *be finite sets. Then we can choose* $\left\{U_{\{e_2\}}\right\}$ *and* $\left\{V_{\{e_2\}}\right\}$ *AS closed sets such that* $\left\{U_{\{e_2\}}\right\} \widetilde{\cap} \left\{V_{\{e_2\}}\right\} \widetilde{=} \widetilde{\varnothing}$ *But for* $A, B \subseteq E$, *we cannot find any AS open sets* $F * A$ *and* $G * B$ *such that* $\left\{U_{\{e_2\}}\right\} \widetilde{\subseteq} F * A$, $\left\{V_{\{e_2\}}\right\} \widetilde{\subseteq} G * B$ *and* $F * A \widetilde{\cap} G * B \widetilde{=} \widetilde{\varnothing}$. *For this purpose, suppose that, there exist* $\left\{(R - U)_{\{e_2\}}\right\}$, $\left\{(R - V)_{\{e_2\}}\right\}$ *be AS open sets such that* $\left\{V_{\{e_2\}}\right\} \widetilde{\subseteq} \left\{(R - U)_{\{e_2\}}\right\}$, $\left\{U_{\{e_2\}}\right\} \widetilde{\subseteq} \left\{(R - V)_{\{e_2\}}\right\}$ *and* $\left\{(R - U)_{\{e_2\}}\right\} \widetilde{\cap} \left\{(R - V)_{\{e_2\}}\right\} \widetilde{=} \widetilde{\varnothing}$. *Therefore, we can obtain* $U \cup V = \mathbb{R}$. *This result contradicts the finite selection of* U *and* V. *Hence,* $\left(\widetilde{\mathbb{R}}, \widetilde{\tau}, E\right)$ *is not an Orhan space and so it is not a* P_4 *space.*

Theorem 5. *Ref [40]. Let* $\left(\widetilde{X}, \widetilde{\tau}, E\right)$ *be a* P_4 *space, then it is also a* P_3 *space.*

Conclusion 1. *Ref [40]. Any AS topological space* $\left(\widetilde{X}, \widetilde{\tau}, E\right)$ *is a* P_4 *space* $\Longrightarrow P_3$ *space* $\Longrightarrow P_2$ *space* $\Longrightarrow P_1$ *space* $\Longrightarrow P_0$ *space.*

3. Results

Monad Metric and Monad Metrizable

We will use $\widetilde{X}$ for amply soft absolute set defined over an initial universe X and a universal parameter set E for the following pages.

Definition 26. *Let* $\left(\widetilde{X}, \widetilde{\tau}, E\right)$ *be an AS topological space and* $a, b \in E$ *such that* $a \neq b$. *If there exist* $x \in X$ *and AS open sets* $F * A$ *and* $G * B$ *such that* $x_a \widetilde{\in} F * A$, $x_b \widetilde{\notin} \overline{F * A}$; *and* $x_b \widetilde{\in} G * B$, $x_a \widetilde{\notin} \overline{G * B}$, *then* x *is called as a passing point between* a *and* b *parameter points.*

Definition 27. $\widetilde{d} : \widetilde{X} \times \widetilde{X} \to \mathbb{R}$ *is said to be a monad metric on the amply soft set* $\widetilde{X}$ *if* $\widetilde{d}$ *satisfies the following conditions:*

1. $\widetilde{d}(x_a, x_b) \geq 0$ *for all* $a, b \in E$ *and* $x \in X$ *is a passing point between* a *and* b,
2. $\widetilde{d}(x_a, x_b) = 0$ *iff* $a = b$, *for all* $a, b \in E$ *and* $x \in X$ *is a passing point between* a *and* b,
3. $\widetilde{d}(x_a, x_b) = \widetilde{d}(x_b, x_a)$ $a, b \in E$ *and* $x \in X$ *is a passing point between* a *and* b,
4. $\widetilde{d}(x_a, x_c) \leq \widetilde{d}(x_a, x_b) + \widetilde{d}(x_b, x_c)$ $a, b, c \in E$ *and* $x \in X$ *is a passing point between* a *and* b; *and between* b *and* c.

Amply soft absolute set $\widetilde{X}$ *with a monad metric* $\widetilde{d}$ *on* $\widetilde{X}$ *is called as a monad metric space and denoted by* $\left(\widetilde{X}, \widetilde{d}\right)$. *The above conditions are said to be monad metric axioms.*

Let us consider the Proposition 3. Let $\left(\widetilde{X}, \widetilde{\tau}, E\right)$ *be an AS topological space and* $a, b, c \in E$. *Let* $\widetilde{d}$ *be a monad metric over* $\widetilde{X}$. *Here* d_a, d_b, d_c *may be defined as metric or discrete metric or pseudo metric in* (X, τ_a), (X, τ_b), (X, τ_c) *topological spaces respectively.*

Definition 28. *Let $\widetilde{d}: \widetilde{X} \times \widetilde{X} \to \mathbb{R}$ be a monad metric on the amply soft set $\widetilde{X}$ and $a, b \in E$ such that $a \neq b$; $x, y, z \in X$ such that $x \neq y \neq z$. Let d_a and d_b be metric or discrete metric or pseudo metric defined over X. Then $\widetilde{d}: \widetilde{X} \times \widetilde{X} \to \mathbb{R}$ is a mapping given by*

$$\widetilde{d}(x_a, x_b) = \begin{cases} \widetilde{d}(x_a, x_b) + \widetilde{d}(x_b, y_b) = \widetilde{d}(x_a, x_b) + d_b(x, y); \\ if\ x\ is\ a\ passing\ point\ between\ a\ and\ b, \\ \widetilde{d}(x_a, z_a) + \widetilde{d}(z_a, z_b) + \widetilde{d}(z_b, y_b) = d_a(x, z) + \widetilde{d}(z_a, z_b) + d_b(z, y); \\ if\ z\ is\ a\ passing\ point\ between\ a\ and\ b. \end{cases} \quad (31)$$

If it is noted here, it is seen that the monad metric gains importance at the passing points. Therefore, it would be sufficient to define the monad metric for passing points especially. It must be defined as a real number greater than zero.

Our claim was to show that the topologies of the dimensions may be different and nonmetric, but even so a monad metric can be defined between the dimensions. Let us define a monad metric on $\left(\widetilde{X}, \widetilde{\tau}, E\right)$ on the Example 4.

Remark 1. *Let $\left(\widetilde{X}, \widetilde{\tau}, E\right)$ be an AS topological space. Then (X, τ_e) are topologies over same universe $X, e \in E$ from the Proposition 3.*

We know that these topologies may be T_0, T_1, T_2, T_3, T_4 spaces. Then pseudo metric $d: X \times X \to \mathbb{R}$ is a mapping given by

$$d(x, y) = \begin{cases} 1 & if\ there\ exist\ A \in \tau\ such\ that\ x \in A, y \notin A \\ 0 & otherwise \end{cases} \quad \text{for } x, y \in X \quad (32)$$

may be defined.

If (X, τ_a) be discrete topological space, then discrete metric $d: X \times X \to \mathbb{R}$ is a mapping given by

$$d(x, y) = \begin{cases} 1 & if\ x \neq y \\ 0 & if\ x = y \end{cases} \quad \text{for } x, y \in X, a \in E \quad (33)$$

may be defined on it.

Example 8. *Let us consider the AS topology $\left(\widetilde{X}, \widetilde{\tau}, E\right)$ on the Example 4. Let $X = \{x, y, z\}$ be a universal set, $E = \{e_1, e_2, e_3\}$ be a parameter set and let $\widetilde{\tau} = \left\{\widetilde{\varnothing}, \widetilde{X}, \left\{\{x, y\}_{\{e_1\}}, \{x, z\}_{\{e_2\}}, \{y, z\}_{\{e_3\}}\right\}, \left\{\{z\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{x\}_{\{e_3\}}\right\}\right\}$. We know that from the Example 6, it is a P_4 space.*

Then, there are $(X, \tau_{e_1}) = \{\varnothing, X, \{x, y\}, \{z\}\}$, $(X, \tau_{e_2}) = \{\varnothing, X, \{x, z\}, \{y\}\}$ and $(X, \tau_{e_3}) = \{\varnothing, X, \{y, z\}, \{x\}\}$ topologies over same universe X from the Proposition 3.

It is clearly seen that (X, τ_{e_1}), (X, τ_{e_2}) and (X, τ_{e_3}) are T_0 spaces. Then, a pseudo metric $d: X \times X \to \mathbb{R}$ is a mapping given by

$$d(x, y) = \begin{cases} 1 & if\ there\ exist\ A \in \tau\ such\ that\ x \in A, y \notin A \\ 0 & otherwise \end{cases} \quad \text{for } x, y \in X \quad (34)$$

can be defined.

Let us find passing points for all $a, b \in E$ such that $a \neq b$. For shortly take $F * A$ and $G * B$ such that $\left\{\{x, y\}_{\{e_1\}}, \{x, z\}_{\{e_2\}}, \{y, z\}_{\{e_3\}}\right\} \widetilde{=} F * A$ and $\left\{\{z\}_{\{e_1\}}, \{y\}_{\{e_2\}}, \{x\}_{\{e_3\}}\right\} \widetilde{=} G * B$ on $\widetilde{\tau}$.

For $e_1, e_2 \in E$; there exist $y \in X$ and AS open sets $F * A$ and $G * B$ such that $y_{e_1} \widetilde{\in} F * A$, $y_{e_2} \widetilde{\notin F * A}$; and $y_{e_2} \widetilde{\in} G * B$, $y_{e_1} \widetilde{\notin G * B}$. So y is a passing point between e_1 and e_2 parameter points, from the Definition 26. Similarly, we can see that there exist $z \in X$ and AS open sets $F * A$ and $G * B$ such that $z_{e_1} \widetilde{\in} G * B$, $z_{e_2} \widetilde{\notin G * B}$; and $z_{e_2} \widetilde{\in} F * A$, $z_{e_1} \widetilde{\notin F * A}$. So z is a passing point between e_1 and e_2 parameter points, from the Definition 26.

For $e_1, e_3 \in E$; there exist $x \in X$ and AS open sets $F*A$ and $G*B$ such that $x_{e_1}\widetilde{\in}F*A$, $x_{e_3}\widetilde{\notin}\overline{F*A}$; and $x_{e_3}\widetilde{\in}G*B$, $x_{e_1}\widetilde{\notin}\overline{G*B}$. So x is a passing point between e_1 and e_3 parameter points, from the Definition 26. Similarly, we can see that there exist $z \in X$ and AS open sets $F*A$ and $G*B$ such that $z_{e_1}\widetilde{\in}G*B$, $z_{e_3}\widetilde{\notin}\overline{G*B}$; and $z_{e_3}\widetilde{\in}F*A$, $z_{e_1}\widetilde{\notin}\overline{F*A}$. So z is a passing point between e_1 and e_3 parameter points, from the Definition 26.

For $e_2, e_3 \in E$; there exist $x \in X$ and AS open sets $F*A$ and $G*B$ such that $x_{e_2}\widetilde{\in}F*A$, $x_{e_3}\widetilde{\notin}\overline{F*A}$; and $x_{e_3}\widetilde{\in}G*B$, $x_{e_1}\widetilde{\notin}\overline{G*B}$. So x is a passing point between e_2 and e_3 parameter points, from the Definition 26. Similarly, we can see that there exist $y \in X$ and AS open sets $F*A$ and $G*B$ such that $y_{e_2}\widetilde{\in}G*B$, $y_{e_3}\widetilde{\notin}\overline{G*B}$; and $y_{e_3}\widetilde{\in}F*A$, $y_{e_2}\widetilde{\notin}\overline{F*A}$. So y is a passing point between e_1 and e_3 parameter points, from the Definition 26.

Therefore,

The passing points on $\left(\widetilde{X}, \widetilde{\tau}, E\right)$ are stated like the following:

- *between e_1 and e_2; passing points y and z;*
- *between e_1 and e_3; passing points x and z;*
- *between e_2 and e_3; passing points x and y.*

Let us define monad metric $\widetilde{d}$ privately for passing points from the Definition 28. Let $\widetilde{d} : \widetilde{X} \times \widetilde{X} \to \mathbb{R}$ be a mapping given by

$$\widetilde{d}(x_a, x_b) = \begin{cases} 5 & if\ a \neq b,\ x \text{ is a passing point between } a \text{ and } b, a, b \in E, x \in X, \\ 0 & if\ a = b,\ x \text{ is a passing point between } a \text{ and } b, a, b \in E, x \in X. \end{cases} \quad (35)$$

It is clearly seen that the monad metric $\widetilde{d}$ provides the monad metric axioms. Then $\widetilde{d} : \widetilde{X} \times \widetilde{X} \to \mathbb{R}$ is a monad metric. Also we can calculate $\widetilde{d}(x_a, y_b)$ for all $x_a, y_b \widetilde{\in} \widetilde{X}$ from the Definition 28. For example,

$$\widetilde{d}(x_{e_1}, z_{e_3}) = \widetilde{d}(x_{e_1}, x_{e_3}) + \widetilde{d}(x_{e_3}, z_{e_3}),\ x \text{ is a passing point and } Definition\ 28 \quad (36)$$

$$= 5 + d_{e_3}(x, z), \quad Definition\ 28. \quad (37)$$

$$= 5 + 1; \quad \text{there exist } \{x\} \in \tau_3 \text{ such that } x \epsilon \{x\},\ z \notin \{x\} \quad (38)$$

$$= 6. \quad (39)$$

$$\widetilde{d}(z_{e_3}, x_{e_1}) = \widetilde{d}(z_{e_3}, x_{e_3}) + \widetilde{d}(x_{e_3}, x_{e_1}),\ x \text{ is a passing point and } Definition\ 28. \quad (40)$$

$$= d_{e_3}(z, x) + 5;\ Definition\ 28. \quad (41)$$

$$= 1 + 5; \quad \text{there exist } \{x\} \in \tau_3 \text{ such that } x \epsilon \{x\},\ z \notin \{x\} \quad (42)$$

$$= 6. \quad (43)$$

So we can see that $\widetilde{d}(x_{e_1}, z_{e_3}) = \widetilde{d}(z_{e_3}, x_{e_1}) = 6$.

$$\widetilde{d}(x_{e_1}, y_{e_2}) = \widetilde{d}(x_{e_1}, y_{e_1}) + \widetilde{d}(y_{e_1}, y_{e_2}),\ y \text{ is a passing point and } Definition\ 28. \quad (44)$$

$$= d_{e_1}(x, y) + 5; \quad Definition\ 28 \quad (45)$$

$$= 0 + 5;\ \text{there is not any exist } A \in \tau_1 \text{ such that } x \in A,\ y \notin A \quad (46)$$

$$= 5. \quad (47)$$

$$\widetilde{d}(y_{e_2}, z_{e_3}) = \widetilde{d}(y_{e_2}, y_{e_3}) + \widetilde{d}(y_{e_3}, z_{e_3}),\ y \text{ is a passing point and } Definition\ 3.3. \quad (48)$$

$$= 5 + d_{e_3}(y, z); Definition\ 3.3. \quad (49)$$

$$= 5 + 0; \quad ;\ \text{there is not any exist } A \in \tau_3 \text{ such that } x \in A,\ y \notin A \quad (50)$$

$$= 5. \quad (51)$$

So we can see that $\widetilde{d}(x_{e_1}, z_{e_3}) \leq \widetilde{d}(x_{e_1}, y_{e_2}) + \widetilde{d}(y_{e_2}, z_{e_3})$ *it satisfies triangular inequality.*

$$6 \leq 5 + 5 \tag{52}$$

$$6 \leq 10. \tag{53}$$

Definition 29. *An AS topological space* $\left(\widetilde{X}, \widetilde{\tau}, E\right)$ *is said to be a monad metrizable if a monad metric* $\widetilde{d}$ *can be defined on* $\widetilde{X}$ *such that the AS topology induced by* $\widetilde{d}$ *is* $\widetilde{\tau}$*. Otherwise, the AS space* $\widetilde{X}$ *is called as a monad non-metrizable.*

Theorem 6. *Any* P_4 *space is also a monad metrizable space.*

Proof 1. Let $\left(\widetilde{X}, \widetilde{\tau}, E\right)$ be a P_4 AS topological space and $A, B, C, D, Z \subset E$. Since $\left(\widetilde{X}, \widetilde{\tau}, E\right)$ is an AS topological space, (X, τ_e) are topologies over same universe X from the Proposition 3. Then from the Remark 1. one of metric, pseudo metric, discrete metric, etc., may be defined on these topologies. Then we want to learn if $\left(\widetilde{X}, \widetilde{\tau}, E\right)$ have got passing points for all $e \in E$ from the Definition 28. Since $\left(\widetilde{X}, \widetilde{\tau}, E\right)$ is a P_4 space, it is both an Orhan space and a P_1 space from the Definition 25. Let $H * C$ be an AS closed set and $F * A$ be an AS open set such that $H * C \widetilde{\subseteq} F * A$. Because $F * A$ is an AS open set, $\widetilde{X \backslash F * A}$ is an AS closed set from the Definition 16. Also, because of $H * C \widetilde{\subseteq} F * A$, $H * C \widetilde{\cap} \left(\widetilde{X \backslash F * A}\right) \widetilde{=} \widetilde{\varnothing}$. Since $\left(\widetilde{X}, \widetilde{\tau}, E\right)$ is an Orhan space, there exist $K * D$ and $W * Z$ AS open sets such that $H * C \widetilde{\subseteq} K * D$ and $\left(\widetilde{X \backslash F * A}\right) \widetilde{\subseteq} W * Z$ and $K * D \widetilde{\cap} W * Z \widetilde{=} \widetilde{\varnothing}$ from the Definition 24. Now we show if $\overline{K * D \widetilde{\cap} W * Z} \widetilde{=} \widetilde{\varnothing}, a \in E, x \in X$. Let $x_a \widetilde{\in} \overline{K * D \widetilde{\cap} W * Z}$. Then $x_a \widetilde{\in} \overline{K * D}$ and $x_a \widetilde{\in} W * Z$. Then $K * D \widetilde{\cap} W * Z \widetilde{\neq} \widetilde{\varnothing}$. This contradicts $K * D \widetilde{\cap} W * Z \widetilde{\neq} \widetilde{\varnothing}$ So $\overline{K * D \widetilde{\cap} W * Z} \widetilde{=} \widetilde{\varnothing}$. Also $\left(\widetilde{X}, \widetilde{\tau}, E\right)$ is a P_1 space, there exist $a, b \in E$ such that $a \neq b$ and there exist $x \in X$ and AS open sets $K * D$ and $W * Z$ such that $x_a \widetilde{\in} K * D$ and $x_b \widetilde{\notin} K * D$; and $x_b \widetilde{\in} W * Z$ and $x_a \widetilde{\notin} W * Z$. Because $\overline{K * D \widetilde{\cap} W * Z} \widetilde{=} \widetilde{\varnothing}$ and from similar way we can say that $K * D \widetilde{\cap} \overline{W * Z} \widetilde{=} \widetilde{\varnothing}$. Then, there exist $x_a \widetilde{\in} K * D$ and $x_b \widetilde{\notin} \overline{K * D}$; and $x_b \widetilde{\in} \overline{W * Z}$ and $x_a \widetilde{\notin} \overline{W * Z}$. So x is a passing point between a and b parameter points. This is true for all $a, b \in E$ and then $\left(\widetilde{X}, \widetilde{\tau}, E\right)$ P_4 AS topological space is also a monad metrizable. □

Example 9. *Let us consider the indiscrete PAS topology on the Example 3. Let us see if it is a monad metrizable space. We know that it is a* P_4 *space from the Example 6. We know that any* P_4 *AS topological space is a monad metrizable space from the Theorem 6. So, it is monad metrizable space. Let us see it:*

Let $e_1, e_2 \in E$ *such that* $e_1 \neq e_2$*. There exist* $x \in X$ *and AS open sets* $\left\{X_{\{e_1\}}\right\}$ *and* $\left\{X_{\{e_2\}}\right\}$ *such that* $x_{e_1} \widetilde{\in} \left\{X_{\{e_1\}}\right\}$ *and* $x_{e_2} \widetilde{\notin} \overline{\left\{X_{\{e_1\}}\right\}} \left(= \left\{X_{\{e_1\}}\right\}\right)$*; and* $x_{e_2} \widetilde{\in} \left\{X_{\{e_2\}}\right\}$ *and* $x_{e_1} \widetilde{\notin} \overline{\left\{X_{\{e_2\}}\right\}} \left(= \left\{X_{\{e_2\}}\right\}\right)$ *so* x *is a passing point between* e_1 *and* e_2 *parameter points for all* $x \in X$*. Also we know that* (X, τ_1) *and* (X, τ_2) *indiscrete topological spaces and pseudo metrics can be defined on them like* $d_e : X \times X \to \mathbb{R}$ *is a mapping given by*

$$d(x, y) = \begin{cases} 1 & if \text{ there exist } A \in \tau \text{ such that } x \in A, y \notin A \\ 0 & \text{otherwise} \end{cases} \quad \text{for } x, y \in X. \tag{54}$$

For example: let $x, y, z \in X$ *and* $e_1, e_2 \in E$*. We know that* $\forall x \in X$ *is a passing point between* e_1 *and* e_2 *from the above explanation. Let us define monad metric* $\widetilde{d}$ *privately for passing points from the Definition 28. Let* $\widetilde{d} : \widetilde{X} \times \widetilde{X} \to \mathbb{R}$ *be a mapping given by*

$$\widetilde{d}(x_{e_1}, x_{e_2}) = \begin{cases} 1 & if\ e_1 \neq e_2,\ x \text{ is a passing point between } e_1 \text{ and } e_2; e_1, e_2 \in E, x \in X, \\ 0 & if\ e_1 = e_2,\ x \text{ is a passing point between } e_1 \text{ and } e_2; e_1, e_2 \in E, x \in X. \end{cases} \tag{55}$$

Let us calculate $\widetilde{d}(x_{e_1}, y_{e_2})$

$$\widetilde{d}(x_{e_1}, y_{e_2}) = \widetilde{d}(x_{e_1}, x_{e_2}) + \widetilde{d}(x_{e_2}, y_{e_2}), \text{ x is a passing point between } e_1 \text{ and } e_2; \text{ and Definition 28.}$$

$$= 1 + d_{e_2}(x, y), \text{ Definition 28.} \tag{56}$$

$$= 1 + 0, \text{ from pseodo metric } d_e = 1. \tag{57}$$

Remark 2. *Let $d : X \times X \to \mathbb{R}$ be a metric and (X, d) be a metric space with d, r be a non-negative real number, $x, y \in X$. We know that $B(x, r) = \{x \in X : d(x, y) < r\}$ is an open ball with centre x and radius r. Namely, for $a \in E$, $d_a : X \times X \to \mathbb{R}$ be a metric and (X, d_a) a be metric space with d_a, r be a non-negative real number, x, $y \in X$. So $B_a(x, r) = \{x \in X : d_a(x, y) < r\}$ is an open ball with centre x and radius r. Similarly, $B_a[x, r] = \{x \in X : d_a(x, y) \leq r\}$ is a closed ball with centre x and radius r.*

Definition 30 *Let (X, d_n) be metric spaces over same universe X, $n \in \mathbb{N}, \mathbb{N}$ is a natural numbers and (X, τ_n) be their topologies such that they are created by open balls from these metrics. Let $\left(\widetilde{X}, \widetilde{\tau}, E\right)$ be their PAS topology. It is called as a PAS metric space.*

It is noted here that a PAS metric space is not any metric space also any monad metric space, it is an AS topological space.

Theorem 7. *Any PAS metric space is also a monad metrizable space.*

Proof 2. Let $\left(\widetilde{X}, \widetilde{\tau}, E\right)$ be a PAS metric space. Then there exist (X, d_a) metric spaces over same universe X, $a \in E$ from the Definition 30.

(X, d_a) are metric spaces from the Remark 2. We want to learn if $\left(\widetilde{X}, \widetilde{\tau}, E\right)$ have got passing points for all $e \in E$ from the Definition 28. For $a, b \in E$ such that $a \neq b$, r is a non-negative real number and there exist $x \in X$, $\{B_a(x, r)\}$ and $\{B_b(x, r)\}$ AS open sets such that $x_a \widetilde{\in} \{B_a(x, r)\}$ and $x_b \widetilde{\notin} \{B_b(x, r)\}$; and $x_b \widetilde{\in} \{B_b(x, r)\}$ and $x_a \widetilde{\notin} \{B_b[x, r]\}$. (it is noted here that $\{B_a[x, r]\}$ is an AS closed set because $\widetilde{X / \{B_a[x, r]\}}$ is an AS open set on $\widetilde{\tau}$ and similarly $\{B_b[x, r]\}$ is an AS closed set too). So x is a passing point between a and b parameter points. This is similar for all $a, b \in E$ such that $a \neq b$ and there exist $x \in X$. Then $\left(\widetilde{X}, \widetilde{\tau}, E\right)$ is a monad metrizable space. □

Example 10. *Let $\mathbb{R}$ be real numbers and $(\mathbb{R}, d_1)$, $(\mathbb{R}, d_2)$ and $(\mathbb{R}, d_3)$ be metric spaces over same universe $\mathbb{R}$ and*

$$d_1 : \mathbb{R} \times \mathbb{R} \to \mathbb{R}, d_1(x, y) = \left|x - y\right| \tag{58}$$

$$d_2 : \mathbb{R} \times \mathbb{R} \to \mathbb{R}, d_2(x, y) = \left|\frac{x}{1 + |x|} - \frac{y}{1 + \left|y\right|}\right| \tag{59}$$

and

$$d_3 : \mathbb{R} \times \mathbb{R} \to \mathbb{R}, d_3(x, y) = \begin{cases} 1 & if\ x \neq y, \\ 0 & if\ x = y. \end{cases} \tag{60}$$

be their metrics respectively. Let $\left(\widetilde{\mathbb{R}}, \widetilde{\tau}, E\right)$ be their PAS metric space over $\mathbb{R}$, $E = \{a, b, c\}$ with elements of it respectively with a mapping $f : \{1, 2, 3\} \to \{a, b, c\}$. Then $\left(\widetilde{\mathbb{R}}, \widetilde{\tau}, E\right)$ is a monad metrizable space from the Theorem 7. Let $3 \in \mathbb{R}$ be a passing point between a and b parameter points, $a, b \in E$; $5 \in \mathbb{R}$ be a passing point between b and c parameter points, $b, c \in E$. Therefore, we can define monad metric $\widetilde{d}$.

Let $\widetilde{d} : \widetilde{\mathbb{R}} \times \widetilde{\mathbb{R}} \to \mathbb{R}$ be a mapping given by

$$\widetilde{d}(3_a, 3_b) = \begin{cases} 5 & if\ a \neq b, 3 \in \mathbb{R} \text{ is a passing point between a and b; } a, b \in E, \\ 0 & if\ a = b, 3 \in \mathbb{R} \text{ is a passing point between a and b; } a, b \in E. \end{cases} \tag{61}$$

$$\widetilde{d}(5_b, 5_c) = \begin{cases} 3 & if\ b \neq c, 5 \in \mathbb{R} \text{ is a passing point between a and b; } b, c \in E, \\ 0 & if\ b = c, 5 \in \mathbb{R} \text{ is a passing point between a and b; } b, c \in E. \end{cases} \tag{62}$$

So $\left(\widetilde{\mathbb{R}}, \widetilde{\tau}, E\right)$ *is a monad metrizable space from the Theorem 7. Then we can calculate the followings.*

(a) $\widetilde{d}(3_a, 5_b)$
(b) $\widetilde{d}(6_a, 9_a)$
(c) $\widetilde{d}(8_b, 5_c)$
(d) $\widetilde{d}(2_b, 5_b)$
(e) $\widetilde{d}(2_a, 7_c)$

A. $\widetilde{d}(3_a, 5_b) = \widetilde{d}(3_a, 3_b) + \widetilde{d}(3_b, 5_b)$ $3 \in \mathbb{R}$ *is a passing point between a and b;* $a, b \in E$,

$$= 5 + d_b(3,5) = 5 + \left|\frac{3}{1+3} - \frac{5}{1+5}\right| = 5 + \frac{1}{12} = \frac{61}{12}. \tag{63}$$

B. $\widetilde{d}(6_a, 9_a) = d_a(6,9)$ *from the Definition 28.*

$$= |6-9| = 3. \tag{64}$$

C. $\widetilde{d}(8_b, 5_c) = \widetilde{d}(8_b, 5_b) + \widetilde{d}(5_b, 5_c)$, $5 \in \mathbb{R}$ *is a passing point between b and c;* $b, c \in E$, $= d_b(8,5) + 3$ *from the Definition 28.* $= \left|\frac{8}{1+8} - \frac{5}{1+5}\right| + 3 = \frac{1}{18} + 3 = \frac{55}{18}$.

D. $\widetilde{d}(2_b, 5_b) = d_b(2,5)$ *from the Definition 28.*

$$= \left|\frac{2}{1+2} - \frac{5}{1+5}\right| = \frac{1}{6}. \tag{65}$$

E. $\widetilde{d}(2_a, 7_c) = \widetilde{d}(2_a, 3_a) + \widetilde{d}(3_a, 3_b) + \widetilde{d}(3_b, 5_b) + \widetilde{d}(5_b, 5_c) + \widetilde{d}(5_c, 7_c)$, $3, 5 \in \mathbb{R}$ *are passing points between a and b; b and c respectively ;* $a, b, c \in E$,

$$= d_a(2,3) + 5 + d_b(3,5) + 3 + d_c(5,7) \tag{66}$$

$$= |2-3| + 5 + \left|\frac{3}{1+3} - \frac{5}{1+5}\right| + 3 + 1 \tag{67}$$

$$= 10 + \frac{1}{12} = \frac{121}{12}. \tag{68}$$

4. Discussions

Do the topologies of each dimension have to be same and metrizable for metricization of space? I have shown that this is not necessary with monad metrizable spaces. For example, a monad metrizable space may have got any indiscrete topologies, discrete topologies, different metric spaces, or any topological spaces in each different dimension. I compute the distance in real space between such topologies. First of all, the passing points between different topologies are defined. Then monad metric is defined. Then a monad metrizable space and a PAS metric space are presented. I show that any PAS metric space is also a monad metrizable space, and present some properties and some examples about it. A similar situation first attracted the attention of Papadapoulos and Scardigly in their study "Spacetimes as topological spaces and the need to take methods of general topology more seriously." They sought answers to "Why is the manifold topology in a spacetime taken for granted? "Why do we prefer to use Riemann open balls as basic-open sets, while there also exists a Lorentz metric [41]" shaped questions. In short, we should consider new topologies for the space-time dimension. Perhaps "the 11th dimension, which appears when physicists working on string or M theory, explains that the Heterotic-E matching constant is greater than 1 but not less than 1 [1]," it is a different topology. Which reminds me of discrete topology.

5. Conclusions

In my next study, I will study the concept of monad metric spaces, toward which I took the preliminary step in this study. A space concept in monad metric spaces will gain a new perspective. With the monad openings, the concept of openness will emerge from two dimensions and will become a gateway between the dimensions. I think that this and my future work will provide researchers with concepts and new pursuits in this context.

Funding: This research received no external funding.

Conflicts of Interest: The author declares no conflict of interest.

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