ISSN 1995-0802, Lobachevskii Journal of Mathematics, 2023, Vol. 44, No. 11, pp. 4913–4919. © Pleiades Publishing, Ltd., 2023.

Approximation Results by Statistical Convergence Based on a Power Series in Modular Spaces

E. Tas[1]* and T. Yurdakadim[2]**

(Submitted by A. I. Volodin)

[1]Kırşehir Ahi Evran University, Department of Mathematics, Kırşehir, 40100 Turkey

[2]Bilecik Şeyh Edebali University, Department of Mathematics, Bilecik, 11100 Turkey

Received August 14, 2023; revised September 11, 2023; accepted September 15, 2023

Abstract—In this study, we present some approximation results in modular spaces for positive linear operators with the use of P-statistical convergence which is recently added to literature by combining statistical convergence and power series. As an application, we provide an example which shows that our theorems are efficient to use since P-statistical convergence assigns a limit to a divergent sequence. It is noteworthy to express that a sequence can be statistically convergent without being P-statistically convergent and vice versa.

DOI: 10.1134/S1995080223110379

Keywords and phrases: *Korovkin type approximation, power series method, statistical convergence, modular spaces.*

1. PRELIMINARIES

Korovkin theory enables a very simple and useful criterion in order to approximate a function by nicer and simpler functions and its motivation depends on the proof of Bernstein which is a smart alternative proof of Weierstrass Theorem. Explicitly, Weierstrass Theorem expresses that algebraic polynomials are dense in the space of continuous functions and Korovkin has extended Weierstrass's ideas by formulating a generalization known as Korovkin Theorem. Korovkin's work further depends our understanding of approximation theory on the uniform convergence of positive linear operators in $C[a,b]$, the space of continuous real valued functions defined on $[a,b]$, by testing the convergence only for $\{1,x,x^2\}$ [18]. This theorem is a fundamental result in the theory of approximation which aims to find simpler functions that can closely approximate more complicated ones. Until 2002, various analogs of Korovkin theorem have been studied in [1, 4, 13, 16, 20]. In the approximation theory it is effective to use weaker convergence methods than ordinary convergence in order to get more general and strong results. Actually such attempts have been initiated by Gadjiev and Orhan in [15] with the use of statistical convergence. Later this process has been improved and many generalizations have been stated in this direction [2, 12, 17, 21, 23, 24]. One of these directions is using the P-statistical convergence [3, 10, 11] since it overcomes the lack of convergence.

In this paper, we extend some earlier results in modular spaces via P-statistical convergence [24]. We also give an example to show that our Theorems 1 and 2 are stronger than Theorems 3.2 and 3.3 in [4] respectively. The effect of P-statistical convergence is already shown by providing such examples which may converge in P-statistical sense but may not be statistically converge and vice versa [24].

First of all, let us recall P-statistical convergence which is the main tool of this study.

*E-mail: emretas86@hotmail.com

**E-mail: tugbayurdakadim@hotmail.com

Let (p_j) be a sequence of reals with $p_0 > 0$ and $p_1, p_2, ... \geq 0$ with $p(t) := \sum_{j=0}^{\infty} p_j t^j$ and radius of convergence $R \in (0, \infty]$. If the limit

$$\lim_{t \to R^-} \frac{1}{p(t)} \sum_{j=0}^{\infty} s_j p_j t^j = l$$

exists, then it is called that $s = (s_j)$ converges to l in power series sense [19, 22]. The regularity of power series is checked with this limit

$$\lim_{t \to R^-} \frac{p_j t^j}{p(t)} = 0, \quad \text{for each} \quad j \in \mathbb{N}_0.$$

P-statistical convergence is a novel extension of ordinary convergence [9], i.e., a sequence can be P-statistically convergent but not ordinary convergent. The idea behind it is combining statistical convergence and power series in order to obtain a powerful, incompatible concept of convergence. Let us recall its definition.

Definition 1 [24]. Let P be regular and $K \subset \mathbb{N}_0$. If

$$\delta_P(K) := \lim_{t \to R^-} \frac{1}{p(t)} \sum_{j \in K} p_j t^j$$

exists, then $\delta_P(K)$ is called the P-density of K. The sequence $s = (s_j)$ is P-statistically convergent to l if for every $\varepsilon > 0$, $\delta_P(K_\varepsilon) = 0$, that is

$$\lim_{t \to R^-} \frac{1}{p(t)} \sum_{j \in K_\varepsilon} p_j t^j = 0,$$

where $K_\varepsilon = \{j \in \mathbb{N}_0 : \ |s_j - l| \geq \varepsilon\}$.

Now we define the concepts of P-statistical limit superior and inferior similarly as in [14] which is another important tool in summability theory. There are also many studies on statistical limit points, cluster points and their relationships. These concepts have already been extended by A-statistical convergence, uniform statistical convergence and ideals.

Definition 2. Let $s = (s_j)$ be a sequence in $\mathbb{R}$ and $K_s = \{b \in \mathbb{R} : \ \delta_P\{j : \ s_j > b\} \neq 0\}$. *$P$-statistical limit superior* is defined by

$$st_P - \limsup s = \begin{cases} \sup K_s, & K_s \neq \emptyset, \\ -\infty, & K_s = \emptyset. \end{cases}$$

Similarly let $L_s = \{a \in \mathbb{R} : \ \delta_P\{j : \ s_j < a\} \neq 0\}$. *$P$-statistical limit inferior* is defined by

$$st_P - \liminf s = \begin{cases} \inf L_s, & L_s \neq \emptyset, \\ \infty, & L_s = \emptyset. \end{cases}$$

Note that we understand from $\delta_P(K) \neq 0$ that either $\delta_P(K) > 0$ or $\delta_P(K)$ does not exist. We also assume the regularity of the power series method P along the paper.

2. KOROVKIN-TYPE RESULTS IN MODULAR SPACES

The notions about the modular have been deeply discussed in [4–8]. However we believe that it would be convenient to recall these concepts here. Consider the bounded interval $I = [a, b] \subset \mathbb{R}$ donated by Lebesgue measure and let $X(I)$ be the space of all real valued measurable functions on I with equality almost everywhere, and $C^\infty(I)$ be the space of all infinitely differentiable functions on I.

A functional $\rho : X(I) \to [0, \infty]$ is a modular on $X(I)$ if

(i) $\rho[f] = 0$ iff $f = 0$ a.e. on I,

(ii) $\rho[-f] = \rho[f]$ for any $f \in X(I)$,

(iii) $\rho[mf + ng] \leq \rho[f] + \rho[g]$ for any $f, g \in X(I)$ and for each $m, n \geq 0$ such that $m + n = 1$.

It is said that ρ is Q-quasi convex provided that there exists a constant $Q \geq 1$ such that the inequality

$$\rho[mf + ng] \leq Qm\rho[Qf] + Qn\rho[Qg]$$

holds for any $f, g \in X(I)$, $m, n \geq 0$ such that $m + n = 1$, in case of $Q = 1$, then ρ is convex, if there exists a constant $Q \geq 1$ such that $\rho[mf] \leq Qm\rho[Qf]$ holds for any $f \in X(I)$, $f \geq 0$ and $m \in (0, 1]$, then ρ is Q-quasi semiconvex. Easily, one can see that Q-quasi convexity implies Q-quasi semiconvexity. Also ρ is monotone if $\rho[f] \leq \rho[g]$ for every $f, g \in X(G)$ with $|f| \leq |g|$.

Here are the most significant subspaces of $X(I)$ in terms of ρ:

$$L^{\rho}(I) := \{f \in X(I) : \lim_{\alpha \to 0^+} \rho[\alpha f] = 0\},$$

$$E^{\rho}(I) := \{f \in L^{\rho}(I) : \rho[\alpha f] < \infty \text{ for all } \alpha > 0\}.$$

It is said that $L^{\varrho}(I)$ is the space generated by ρ and $E^{\rho}(I)$ is the space of the finite elements of $L^{\varrho}(I)$. If ρ is Q-quasi semiconvex, then

$$\{f \in X(I) : \rho[\alpha f] < \infty \text{ for some } \alpha > 0\} = L^{\rho}(I).$$

If there exists a constant $H > 0$ such that $\rho[2t] \leq H\rho[t]$ holds for every $t \geq 0$, then we say that ρ satisfies the Δ_2-condition. Also for a modular ρ, we say

- finite if the characteristic function related to I is in $L^{\rho}(I)$, i.e., $\chi_I \in L^{\rho}(I)$,
- absolutely finite if ρ is finite and for every $\varepsilon > 0$, $\alpha > 0$ there exists $\delta > 0$ such that $\rho[\alpha\chi_K] < \varepsilon$ for every measurable subset $K \subset I$ with $|K| < \delta$,
- strongly finite if $\chi_I \in E^{\rho}(I)$,
- absolutely continuous if there is a positive constant m satisfying the following: for every $f \in X(I)$ with $\rho[f] < \infty$, the following condition holds: for every $\varepsilon > 0$ there is a $\delta > 0$ such that $\rho[mf\chi_B] < \varepsilon$ for every measurable $B \subset I$ with $|B| < \delta$.

$C(I) \subset L^{\varrho}(I)$ holds in the case of ρ is monotone and finite [4]. Similarly, $C(I) \subset E^{\rho}(I)$ in the case of ρ is monotone and strongly finite.

Now we consider a set D with $C^{\infty}(I) \subset D \subset L^{\varrho}(I)$ and also $T := (T_j)$ be a sequence of positive linear operators from D into $X(I)$ such that $C^{\infty}(I) \subset X_T \subset D$ and for any $h \in X_T$, $\lambda > 0$ and a constant $H > 0$

$$st_P - \limsup \rho[\lambda(T_j h)] \leq H\rho[\lambda h]. \tag{1}$$

Theorem 1. *If $st_P - \lim \rho[\lambda(T_j F_v - F_v)] = 0$, for every $\lambda > 0$, holds for the positive linear operators T_j from D into $X(I)$ satisfying condition (1), where $F_v(x) = x^v$ $(v = 0, 1, 2)$, ρ is monotone, strongly finite, absolutely continuous, Q-quasi semiconvex, then for any $f \in L^{\rho}(I)$ such that for any $g \in C^{\infty}(I)$, $f - g \in X_T$, we get $st_P - \lim \rho[\lambda_0(T_j f - f)] = 0$, for some $\lambda_0 > 0$.*

Proof. Initially we show that

$$st_P - \lim \varrho[\mu(T_j g - g)] = 0 \tag{2}$$

for any $g \in C(I)$ and $\mu > 0$. In order to prove it let $g \in C(I)$ and $\mu > 0$. Since g is uniformly continuous and T_j are positive linear operators, one can write for every $\varepsilon > 0$ there exists $\delta > 0$ such that

$$|T_j(g; x) - g(x)| \leq \varepsilon T_j(F_0; x) + c|T_j(F_0; x) - F_0(x)| + \frac{2c}{\delta^2} T_j((y - x)^2; x)$$
$$\text{for any} \quad x \in I \quad \text{and} \quad j \in \mathbb{N}_0,$$

where $c = \sup_{x \in I} |g(x)|$. By using the above inequality, we have

$$|T_j(g; x) - g(x)| \leq \varepsilon + \left(\varepsilon + c + \frac{2c\beta^2}{\delta^2}\right)|T_j(F_0; x) - F_0(x)|$$

$$+\frac{4c\beta}{\delta^2}|T_j(F_1;x)-F_1(x)|+\frac{2c}{\delta^2}|T_j(F_2;x)-F_2(x)|,$$

where $\beta=\max\{|a|,|b|\}$. Indeed for every $\mu>0$, we obtain

$$\mu|T_j(g;x)-g(x)|\leq \mu\varepsilon+K\mu\{|T_j(F_0;x)-F_0(x)|+|T_j(F_1;x)-F_1(x)|+|T_j(F_2;x)-F_2(x)|\}$$

where $K=\max\{\varepsilon+c+\frac{2c\beta^2}{\delta^2},\frac{4c\beta}{\delta^2},\frac{2c}{\delta^2}\}$. Since ρ is monotone, from the last inequality we get

$$\rho[\mu|T_j(g)-g|]\leq\rho[\mu\varepsilon+K\mu|T_jF_0-F_0|+K\mu|T_jF_1-F_1|+K\mu|T_jF_2-F_2|]$$

and hence we obtain

$$\rho[\mu|T_j(g)-g|]\leq\rho[4\mu\varepsilon]+\rho[4K\mu|T_jF_0-F_0|]+\rho[4K\mu|T_jF_1-F_1|]+\rho[4K\mu|T_jF_2-F_2|].$$

By Q-quasi semiconvexity and strong finiteness of ρ and supposing $0<\varepsilon\leq 1$, we can write

$$\rho[\mu|T_j(g)-g|]\leq Q\varepsilon\rho[4\mu Q]+\rho[4K\mu|T_jF_0-F_0|]+\rho[4K\mu|T_jF_1-F_1|]+\rho[4K\mu|T_jF_2-F_2|].$$

On the other hand, for $r>0$ we can choose $\varepsilon\in(0,1]$ with $Q\varepsilon\varrho[4\mu Q]<r$. Also consider the sets

$$I_\mu:=\{j:\ \rho[\mu|T_jg-g|]\geq r\},$$
$$I_{\mu,1}:=\left\{j:\ \rho[4\mu K|T_jF_0-F_0|]\geq\frac{r-Q\varepsilon\rho[4\mu Q]}{3}\right\},$$
$$I_{\mu,2}:=\left\{j:\ \rho[4\mu K|T_jF_1-F_1|]\geq\frac{r-Q\varepsilon\rho[4\mu Q]}{3}\right\},$$
$$I_{\mu,3}:=\left\{j:\ \rho[4\mu K|T_jF_2-F_2|]\geq\frac{r-Q\varepsilon\rho[4\mu Q]}{3}\right\}.$$

We can easily show that $I_\mu\subset I_{\mu,1}\cup I_{\mu,2}\cup I_{\mu,3}$. Hence we obtain $\delta_P(I_\mu)\leq\delta_P(I_{\mu,1})+\delta_P(I_{\mu,2})+\delta_P(I_{\mu,3})$. According to the hypothesis, for every $\varepsilon>0$, every $\lambda>0$ and $v=0,1,2$, we get $\delta_P(E_{\varepsilon,v})=0$, where $E_{\varepsilon,v}=\{j\in\mathbb{N}_0:\ \rho[\lambda|T_jF_v-F_v|]\geq\varepsilon\}$. That is $\delta_P(I_{\mu,1})=\delta_P(I_{\mu,2})=\delta_P(I_{\mu,3})=0$ which implies $\delta_P(I_\mu)=0$. This proves (2).

Since $C^\infty(I)\subset C(I)$, (2) holds for any $g\in C^\infty(I)$. Let us take an arbitrary $f\in L^\rho(I)$ such that for any $g\in C^\infty(I)$, $f-g\in X_T$. Since $|I|<\infty$, ρ is strongly finite and absolutely continuous, it is easy to see that ρ is absolutely finite. By using these properties and since $C^\infty(I)$ is modularly dense in $L^\rho(I)$, we get $\{g_j\}\subset C^\infty(I)$ and $\lim_j\rho[3\lambda_0(g_j-f)]=0$ (some $\lambda_0>0$) which implies that for every $\varepsilon>0$ there exists $j_0(\varepsilon)\in\mathbb{N}_0$ such that for each $j\geq j_0$

$$\rho[3\lambda_0(g_j-f)]<\varepsilon. \tag{3}$$

Since T_j is positive and linear, we sketch for every $x\in I$ and $j\in\mathbb{N}_0$

$$\lambda_0|T_j(f;x)-f(x)|\leq\lambda_0|T_j(f-g_{j_0};x)|+\lambda_0|T_j(g_{j_0};x)-g_{j_0}(x)|+\lambda_0|g_{j_0}(x)-f(x)|.$$

We get

$$\rho[\lambda_0(T_jf-f)]\leq\rho[3\lambda_0T_j(f-g_{j_0})]+\rho[3\lambda_0(T_jg_{j_0}-g_{j_0})]+\rho[3\lambda_0(g_{j_0}-f)] \tag{4}$$

with the use of monotonicity of ρ. Also by using (3) and (4), we may write for every $\varepsilon>0$ and $j\geq j_0$

$$\rho[\lambda_0(T_jf-f)]\leq\varepsilon+\rho[3\lambda_0T_j(f-g_{j_0})]+\rho[3\lambda_0(T_jg_{j_0}-g_{j_0})]$$

and then using the facts $g_{j_0}\in C\infty(I)$, $f-g_{j_0}\in X_T$, applying $st_P-\limsup$,

$$st_P-\limsup\rho[\lambda_0(T_jf-f)]\leq\varepsilon+H\rho[3\lambda_0(f-g_{j_0})]+st_P-\limsup\rho[3\lambda_0(T_jg_{j_0}-g_{j_0})]$$
$$\leq\varepsilon(H+1)+st_P-\limsup\rho[3\lambda_0(T_jg_{j_0}-g_{j_0})]\leq\varepsilon(H+1)$$

can be obtained. Because of the fact that ε is an arbitrary positive real number, we get

$$st_P-\limsup\rho[\lambda_0(T_jf-f)]=0.$$

Since $\rho[\lambda_0(T_jf-f)]$ is nonnegative for each $j\in\mathbb{N}_0$, we have

$$st_P-\lim\rho[\lambda_0(T_jf-f)]=0$$

as desired. □

Under Δ_2-condition, we may present the following immediately

Theorem 2. *Let ρ and $T = (T_j)$ satisfy the assumptions of Theorem 1. If ρ satisfies Δ_2-condition, then the followings imply each other*

- $st_P - \lim \rho[\lambda(T_j F_v - F_v)] = 0$ *for any* $\lambda > 0, v = 0, 1, 2$
- $st_P - \lim \rho[\lambda(T_j f - f)] = 0$ *for any* $\lambda > 0$ and $f \in L^\rho(I)$ *such that for any* $g \in C^\infty(I)$, $f - g \in X_T$.

3. APPLICATIONS

Now we present an example which illustrates that our Theorem 1 is stronger than Theorem 3.2 in [4].

Example. Define (p_j) and (s_j) as follows

$$p_j = \begin{cases} 1, & j = 2k, \\ 0, & j = 2k+1, \end{cases} \qquad \text{and} \quad s_j = \begin{cases} 0, & j = 2k, \\ 1, & j = 2k+1. \end{cases}$$

One can observe that P is regular.

Let $I = [0,1]$ and $\phi : [0,\infty) \to [0,\infty)$ be continuous, convex function such that $\phi(0) = 0$, for $u > 0$ $\phi(u) > 0$ and $\lim_{u\to\infty} \phi(u) = \infty$. Then, define a modular by $\rho^\phi : X(I) \to \mathbb{R}$, $\rho^\phi[f] = \int_0^1 \phi(|f(x)|)dx$. The modular space generated by ρ^ϕ is denoted by L^ϕ_ρ. Observe that for every $\varepsilon > 0$, $\delta_P(E_\varepsilon) = 0$, that is (s_j) is P-statistically convergent to 0.

Let us take into consideration the following sequence of positive linear operators $U = (U_j)$ on $L^\phi_\rho[I]$

$$U_j(f;x) = \sum_{k=0}^{j} \binom{j}{k} x^k (1-x)^{j-k} (j+1) \int_{\frac{k}{j+1}}^{\frac{k+1}{j+1}} f(t)dt, \quad x \in [0,1]. \tag{5}$$

By using the above sequence of operators (5), we can define V_j as $V_j(f;x) = (1+s_j)U_j(f;x)$. It has been shown that

$$\rho^\phi[\lambda V_j h] \le (1+s_j)\rho^\phi[2\lambda U_j h] \le (1+s_j) H \rho^\phi[2\lambda h]$$

in [25]. Therefore, we get

$$st_P - \limsup \rho^\phi[\lambda V_j h] \le H\rho^\phi[2\lambda h]$$

for any $h \in X_V := L^\phi_\rho$, every $\lambda > 0$ and for an absolute positive constant P. Since

$$\rho^\phi[\lambda|V_j(F_0) - F_0|] = \rho^\phi[\lambda s_j] = \int_0^1 \phi(\lambda s_j)dx = \phi(\lambda s_j) = s_j\phi(\lambda)$$

we can easily obtain

$$st_P - \lim \rho^\phi[\lambda|V_j(F_0) - F_0|] = st_P - \lim \rho^\phi[\lambda s_j] = st_P - \lim s_j\phi(\lambda) = 0.$$

For the test function F_1, we have that

$$\rho^\phi\left[\lambda|V_j(F_1) - F_1|\right] = \rho^\phi\left[\lambda \left| F_1\left(\frac{j}{j+1} + \frac{js_j}{j+1} - 1\right) + \frac{1}{2(j+1)} + \frac{s_j}{2(j+1)}\right|\right]$$
$$\le \rho^\phi\left[\lambda\left(s_j\frac{2j+1}{2(j+1)} + \frac{3}{2(j+1)}\right)\right] \le s_j\rho^\phi\left[\lambda\frac{2j+1}{j+1}\right] + \rho^\phi\left[\frac{3\lambda}{j+1}\right] \le s_j\rho^\phi[\lambda M] + \rho^\phi\left[\frac{3\lambda}{j+1}\right].$$

By using the last inequality and since ϕ is continuous $\phi(\frac{3\lambda}{j+1}) \to 0$ $(j \to \infty)$ and the fact that P is regular, we have $st_P - \lim \rho^{\phi}[\lambda|V_j(F_1) - F_1|] = 0$. For the test function F_2, we also have that

$$\varrho^{\phi}[\lambda|V_j(F_2) - F_2|] = \rho^{\phi}\left[\lambda\left|F_2\left(\frac{j(j-1)}{(j+1)^2} + \frac{2jF_1}{(j+1)^2} + \frac{1}{3(j+1)^2}\right.\right.\right.$$
$$\left.\left.\left. + s_j\frac{j(j-1)}{(j+1)^2}F_2 + s_j\frac{2jF_1}{(j+1)^2} + s_j\frac{1}{3(j+1)^2} - F_2\right)\right|\right] \le \rho^{\phi}\left[\lambda\left(\frac{15j+4}{3(j+1)^2} + s_j\frac{3j^2+3j+1}{3(j+1)^2}\right)\right]$$
$$\le \rho^{\phi}\left[2\lambda\frac{15j+4}{3(j+1)^2}\right] + \rho^{\phi}\left[2\lambda s_j\frac{3j^2+3j+1}{3(j+1)^2}\right] \le \rho^{\phi}\left[\lambda\frac{30j+8}{3(j+1)^2}\right] + \rho^{\phi}[2\lambda s_j K].$$

Using the same arguments above, we have $st_P - \lim \rho^{\phi}[\lambda|V_j(F_2) - F_2|] = 0$. This shows that (V_j) satisfies hypothesis of our Theorem 1. In the meanwhile (V_j) does not satisfy Theorem 3.2 and Theorem 3.3 in [4] since (s_j) is not ordinary convergent.

ACKNOWLEDGMENTS

The authors would like to thank the reviewers for their valuable comments which improve the quality of the paper.

FUNDING

This work was supported by ongoing institutional funding. No additional grants to carry out or direct this particular research were obtained.

CONFLICT OF INTEREST

The authors of this work declare that they have no conflicts of interest.

REFERENCES

1. F. Altomare and M. Campiti, *Korovkin Type Approximation Theory and Its Applications,* Vol. 7 of *Walter de Gruyter Studies in Mathematics* (de Gruyter, Berlin, 1994).
2. O. G. Atlihan and C. Orhan, "Matrix summability and positive linear operators," Positivity **11**, 387–398 (2007).
3. N. Bayram and S. Yildiz, "Approximation by statistical convergence with respect to power series methods," Hacet. J. Math. Stat. **51**, 1–13 (2022).
4. C. Bardaro and I. Mantellini, "Korovkin theorem in modular spaces," Comment Math. **47**, 239–253 (2007).
5. C. Bardaro, A. Boccuto, X. Dimitriou, and I. Mantellini, "Modular filter convergence theorems for abstract sampling-type operators," Appl. Anal. **92**, 2404–2423 (2013).
6. C. Bardaro and I. Mantellini, "Multivariate moment type operators: Approximation properties in Orlicz spaces," J. Math. Ineq. **2**, 247–259 (2008).
7. C. Bardaro, A. Boccuto, X. Dimitriou, and I. Mantellini, "A Korovkin theorem in multivariate modular function spaces," J. Funct. Spaces Appl. **7**, 105–120 (2005).
8. C. Bardaro, J. Musielak, and G. Vinti, *Nonlinear Integral Operators and Applications,* Vol. 9 of *De Gruyter Series on Nonlinear Analysis and Applications* (Walter de Gruyter, Berlin, 2003).
9. J. Boos, *Classical and Modern Methods in Summability* (Oxford Univ. Press, Oxford, 2000).
10. K. Demirci, S. Yildiz, and F. Dirik, "Approximation via statistical convergence in the sense of power series method of Bögel-type continuous functions," Lobachevskii J. Math. **43**, 2423–2432 (2022).
11. K. Demirci, F. Dirik, and S. Yildiz, "Approximation via statistical relative uniform convergence of sequences of functions at a point with respect to power series method," Afrika Math. **34**, 39 (2023).
12. O. Duman, "A-statistical convergence of sequences of convolution operators," Taiwanese J. Math. **12**, 523–536 (2008).
13. V. K. Dzjadyk, "Approximation of functions by positive linear operators and singular integrals," Mat. Sb. **112**, 508–517 (1966).
14. J. A. Fridy and C. Orhan, "Statistical limit superior and limit inferior," Proc. Am. Mat. Soc. **125**, 3625–3631 (1997).

15. A. D. Gadjiev and C. Orhan, "Some approximation theorems via statistical convergence," Rocky Mountain J. Math. **32**, 129–138 (2002).
16. K. Izuchi, H. Takagi, and S. Watanabe, "Sequential BKW-operators and function algebras," J. Approx. Theory **85**, 185–200 (1996).
17. S. Karakus and K. Demirci, "Matrix summability and Korovkin type approximation theorem on modular spaces," Acta Math. Univ. Comenian. **79**, 281–292 (2010).
18. P. P. Korovkin, *Linear Operators and Approximation Theory* (Hindustan, Delhi, 1960).
19. W. Kratz and U. Stadtmüller, "Tauberian theorems for J_p-summability," J. Math. Anal. Appl. **139**, 362–371 (1989).
20. P. Renaud, "A Korovkin theorem for abstract Lebesgue spaces," J. Math. Anal. Appl. **139**, 362–371 (1989).
21. I. Sakaoglu and C. Orhan, "Strong summation process in L_p spaces," Nonlin. Anal. **86**, 89–94 (2013).
22. U. Stadtmuller and A. Tali, "On certain families of generalized Nörlund methods and power series methods," J. Math. Anal. Appl. **238**, 44–66 (1999).
23. M. Unver, "Abel transforms of positive linear operators on weigthed spaces," Bull. Belg. Math. Soc. Simon Stevin **21**, 813–822 (2014).
24. M. Unver and C. Orhan, "Statistical convergence with respect to power series methods and applications to approximation theory," Numer. Funct. Anal. Optim. **40**, 535–547 (2019).
25. T. Yurdakadim, "Some Korovkin type results via power series method in modular spaces," Commun. Fac. Sci. Univ. Ank. Ser. A1 Math. **65**, 65–76 (2016).