

# Performance analysis and ecological optimization of an irreversible quantum heat engine with spin-1/2 system

Emin Açıkkalp<sup>a,\*</sup>, Mohammad H. Ahmadi<sup>b</sup>

<sup>a</sup> Department of Mechanical Engineering, Engineering Faculty, Bilecik S.E. University, Bilecik, Turkey

<sup>b</sup> Faculty of Mechanical Engineering, Shahrood University of Technology, Shahrood, Iran

## ARTICLE INFO

### Keywords:

Quantum heat engine  
Irreversibility  
Ecological function  
Spin-1/2 system

## ABSTRACT

This paper is about an irreversible quantum spin-1/2 heat engine. Performance of the irreversible quantum heat engine including basic thermodynamic parameters, power output and energy efficiency, are considered, besides ecological function. Ecological function gives a possibility a comparison between power output and exergy destruction. The results show that power output and ecological function have maximum (optimum) points for different effectiveness values which are 0.9 and 0.8 and according to different magnetic field values. All parameters are compared with each other and it is found that the most convenient operation conditions are obtained at optimum ecological function.

## 1. Introduction

Finite time thermodynamics (FTT) began with Curzon-Ahlborn-Novikov engine [1,2]. Purpose of the FTT is to analyze actual heat engines that include irreversibilities. FTT can be combined with micro and nano thermal cycles. In last decades, quantum thermodynamics, which investigates heat and work interactions for the quantum mechanical systems, has been gained attention. In the quantum scale, law of the classical thermodynamic cannot be used and quantum mechanical generalization is an obligation in this scale. In addition, nano technology has been increased importantly and thermodynamic evaluation and design of nano systems begin to attract by using quantum thermodynamics. In the literature, some examples of quantum heat engines [3–40] or heat pumps and refrigerators [41–63] can be found.

Ecological function was presented Angulo-Brown [64] and improved by Yan [65]. This function is described as extracting exergy destruction from power output and it provides a balance between them. While optimizing this function, most convenient operating conditions that are environmental friendly can be determined for any thermal cycle.

In this paper, an irreversible quantum heat engine with spin-1/2 system is investigated. All temperatures in the system is defined as a function of heat source and heat sink temperatures, effectiveness, time periods, magnetic field and friction constant. Besides the basic thermodynamic parameters like power output and energy efficiency, besides the ecological function which is considered to determine most efficient or environmentally friendly conditions. Operation conditions

are investigated and compared for considered parameters. Results are presented and discussed. Finally, most convenient conditions are presented.

## 2. Analysis

Quantum heat engine investigated in this paper is shown in Fig. 1. In the analyses applied in this paper, heat source and heat sink are assumed as infinite and they are kept at constant temperature. System is contacts with hot and cold reservoirs at constant temperature. Heat input to the system and heat output from the system occur in constant pressure. In addition, heat exchanges of the system are supposed to be similar to those via heat exchanger process. That's why effectiveness is used for the heat transfer processes and the system is irreversible. The temperature  $\beta$  (throughout this paper “temperature” will refer to rather than  $T$  for simplicity, if not stated otherwise and  $\beta = 1/k_B T$ , where  $T$  is the absolute temperature) of the working fluid is always well established for given  $S$  and  $\omega$  because, for a two-level system, it is always well defined by the following relation (Expectation value of spin operator) [27]:

$$S = -\frac{1}{2} \tanh\left(\frac{\omega\beta}{2}\right), \quad (1)$$

where  $k_B$  is the Boltzmann coefficient and  $\omega$  magnetic field. For facilitating in calculations, Boltzmann coefficient and Planck constant assumes as 1. Consider a two-level system with a one-parameter Hamiltonian  $H(\omega)$  such that the energy levels are  $-\omega/2$  and  $\omega/2$ , for

\* Corresponding author.

E-mail address: [emin.acikkalp@bilecik.edu.tr](mailto:emin.acikkalp@bilecik.edu.tr) (E. Açıkkalp).

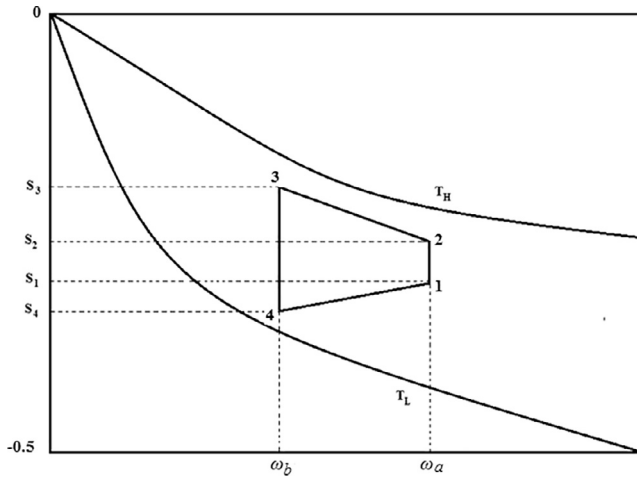


Fig. 1. S- $\omega$  diagram of the irreversible quantum cycle.

One can yield Eqs. (6)–(9) that are function of effectiveness, time period, friction coefficient, heat source and heat sink temperatures using Eqs. (1)–(5).

$$\beta_1 = \frac{1}{\omega_a} 2 \text{ArcTanh} \left( \frac{\left( 2m^2(t_a + t_b - \epsilon_L t_b) + \epsilon_H(\epsilon_L - 1)t_a t_b \text{Tanh}\left(\frac{\beta_H \omega_a}{2}\right) - \epsilon_L t_a t_b \text{Tanh}\left(\frac{\beta_L \omega_b}{2}\right) \right)}{((\epsilon_H(\epsilon_L - 1) - \epsilon_L)t_a t_b)} \right) \quad (6)$$

$$\beta_2 = -\frac{1}{\omega_a} 2 \text{ArcTanh} \left( \frac{\epsilon_H t_a t_b \text{Tanh}\left(\frac{\beta_H \omega_a}{2}\right) + (1 - \epsilon_H) 2m^2(t_a + t_b - \epsilon_L t_b) - \epsilon_L t_a t_b \text{Tanh}\left(\frac{\beta_L \omega_b}{2}\right)}{((\epsilon_H(\epsilon_L - 1) - \epsilon_L)t_a t_b)} \right) \quad (7)$$

$$\beta_3 = -\frac{1}{\omega_b} 2 \text{ArcTanh} \left( \frac{2m^2((\epsilon_H - 1)t_a - t_b) + \epsilon_H t_a t_b \text{Tanh}\left(\frac{\beta_H \omega_a}{2}\right) - (1 - \epsilon_H)\epsilon_L t_a t_b \text{Tanh}\left(\frac{\beta_L \omega_b}{2}\right)}{((\epsilon_H(\epsilon_L - 1) - \epsilon_L)t_a t_b)} \right) \quad (8)$$

$$\beta_4 = -\frac{1}{\omega_b} 2 \text{ArcTanh} \left( \frac{2m^2(\epsilon_L - 1)((\epsilon_H - 1)t_a - t_b) + \epsilon_H(\epsilon_L - 1)t_a t_b \text{Tanh}\left(\frac{\beta_H \omega_a}{2}\right) - \epsilon_L t_a t_b \text{Tanh}\left(\frac{\beta_L \omega_b}{2}\right)}{((\epsilon_H(\epsilon_L - 1) - \epsilon_L)t_a t_b)} \right) \quad (9)$$

example a spin-1/2 system, in a magnetic field of intensity  $B$  ( $T$ ), with  $\omega = 2\mu_B B$  and  $\mu_B$  is the Bohr's magneton [31]. In the analysis applied in this paper, heat source and heat sink are assumed as infinite and they are kept at constant temperature. Heat input to the system and heat output from the system occur in isobaric steps. In addition, heat exchanges of the system are supposed to be similar to those via heat exchanger process. That is why effectiveness ( $\epsilon$ ) is used for the heat transfer processes. Firstly, working substance coupled with hot heat source of temperature  $\beta_H$  the for the period  $t_H$  and magnetic field is constant value at  $\omega_a$ . Then, spin angular momentum changes to  $S_1$  to  $S_2$ . Heat input ( $J$ ) is calculated from Eq. (2) [31,32]:

$$\begin{aligned} Q_H &= \frac{\omega_a}{2} \left( \text{Tanh}\left(\frac{\beta_1 \omega_a}{2}\right) - \text{Tanh}\left(\frac{\beta_2 \omega_a}{2}\right) \right) \\ &= \frac{\omega_a}{2} \epsilon_H \left( \text{Tanh}\left(\frac{\beta_1 \omega_a}{2}\right) - \text{Tanh}\left(\frac{\beta_H \omega_a}{2}\right) \right) \end{aligned} \quad (2)$$

Secondly, working substance is isolated from the hot heat source and magnetic field is converted to  $\omega_b$  at  $t_a$  period. In addition, friction is happened in system and this friction leads to angular momentum  $S_2$  to  $S_3$ . Friction is described with a friction coefficient  $m$ .

Thirdly, working substance is coupled with heat sink at temperature  $\beta_L$  for time period  $t_L$ . Spin angular momentum changes  $S_3$  to  $S_4$ , while magnetic field is constant. Rejected heat is:

$$\begin{aligned} Q_L &= \frac{\omega_b}{2} \left( \text{Tanh}\left(\frac{\beta_3 \omega_b}{2}\right) - \text{Tanh}\left(\frac{\beta_4 \omega_b}{2}\right) \right) \\ &= \frac{\omega_b}{2} \epsilon_L \left( \text{Tanh}\left(\frac{\beta_3 \omega_b}{2}\right) - \text{Tanh}\left(\frac{\beta_L \omega_b}{2}\right) \right) \end{aligned} \quad (3)$$

Finally, this period is similar to second one. Working substance is removed from the heat sink for period  $t_b$ . In this period, magnetic field is converted to  $\omega_a$  and spin angular momentum changes to  $S_4$  to  $S_1$ . Using friction and time periods,  $\beta_3$  and  $\beta_4$  can be written as Eqs. (4) and (5) respectively.

$$\beta_3 = \frac{2}{\omega_b} \left( \text{Arctanh} \left( \text{Tanh} \left( \frac{\beta_2 \omega_a}{2} \right) - \frac{2m^2}{t_a} \right) \right) \quad (4)$$

$$\beta_4 = \frac{2}{\omega_b} \left( \text{Arctanh} \left( \text{Tanh} \left( \frac{\beta_1 \omega_a}{2} \right) - \frac{2m^2}{t_b} \right) \right) \quad (5)$$

Cycle periods can be shown in Eqs. (10)–(12) [22]:

$$t_L = \frac{1}{2a\hbar^2 e^{q\beta_L \omega_b} (e^{\beta_L \omega_b} + 1)} \ln \left( \frac{\text{Tanh}\left(\frac{\beta_L \omega_b}{2}\right) - \text{Tanh}\left(\frac{\beta_3 \omega_b}{2}\right)}{\text{Tanh}\left(\frac{\beta_L \omega_b}{2}\right) - \text{Tanh}\left(\frac{\beta_1 \omega_a}{2}\right) - \frac{2m^2}{t_b}} \right) \quad (10)$$

$$t_H = \frac{1}{2a\hbar^2 e^{q\beta_H \omega_a} (e^{\beta_H \omega_a} + 1)} \ln \left( \frac{\text{Tanh}\left(\frac{\beta_H \omega_a}{2}\right) - \text{Tanh}\left(\frac{\beta_1 \omega_a}{2}\right)}{\text{Tanh}\left(\frac{\beta_H \omega_a}{2}\right) - \text{Tanh}\left(\frac{\beta_3 \omega_b}{2}\right) - \frac{2m^2}{t_a}} \right) \quad (11)$$

$$t = t_H + t_L + t_a + t_b \quad (12)$$

where  $a$  and  $q$  are parameters of the heat reservoirs. Work output of the system ( $J$ ) using first law of the thermodynamics is:

$$W = Q_H + Q_L \quad (13)$$

Power output ( $W$ ) is described as:

$$P = \frac{W}{t} \quad (14)$$

Energy efficiency is the ratio of the work output to added heat to the system:

$$\eta = \frac{W}{Q_H} \quad (15)$$

Exergy destruction rate ( $W$ ) is the lost work based on irreversibilities and it can be written as:

$$Ex_D = \frac{T_o(Q_L \beta_L - Q_H \beta_H)}{t} \quad (16)$$

Finally, ecological function ( $W$ ):

$$E = P - Ex_D \quad (17)$$

### 3. Results and discussion

In this section, results are presented and discussed. Some fixed parameters used in calculations are;  $\omega_a = 5$ ,  $\omega_b = 2$ ,  $a = 2$  1/sn,  $q = -0.5$ ,  $t_a = t_b = 0.01$  sn,  $m = 0.0002$ ,  $\beta_H = 0.2$  1/K,  $\beta_L = 1$  1/K,  $\beta_o = 0.9$  1/K.

In Fig. 2, change of power output, energy efficiency and ecological function parameters can be seen with  $\omega_a$  ( $\epsilon_H = \epsilon_L = 0.9$ ). As it seen, power output and ecological function have optimum (maximum)

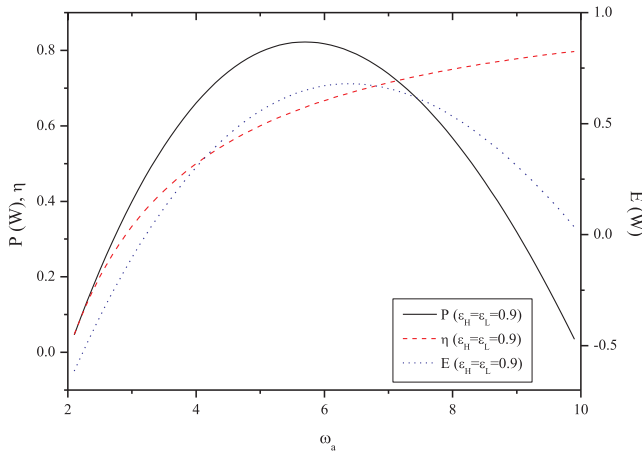


Fig. 2. Change of P, η and E with  $\omega_a$  for  $\epsilon_H = \epsilon_L = 0.9$ .

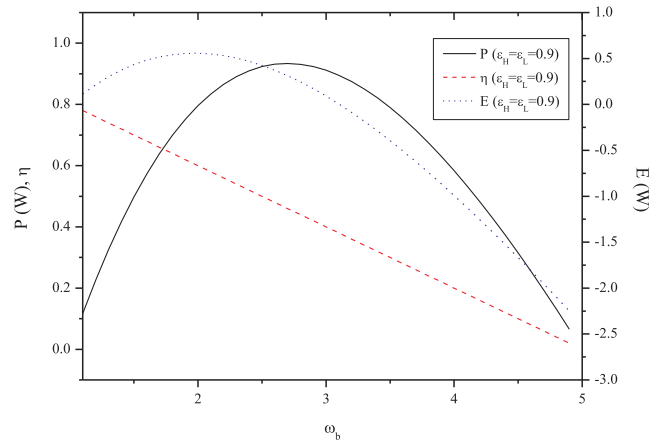


Fig. 4. Change of P, η and E with  $\omega_b$  for  $\epsilon_H = \epsilon_L = 0.9$ .

points. They increase until this point and then they start to decrease. Maximum point of the power output is obtained at 5.7 with 0.822 W and maximum point of the ecological function is obtained at 6.4 with 0.680 W. Energy efficiency is nearly linear and it increases continuously. Its maximum value reaches 0.797 at the  $\omega_a = 9.9$ . In Fig. 3, power output, energy efficiency and ecological function parameters can be seen for  $\omega_a$  ( $\epsilon_H = \epsilon_L = 0.8$ ). According to figure, power output and ecological function have close values and both of them have maximum point. The maximum point of the power output is acquired at 5.7 and the maximum point of the ecological function is acquired at 6.5. Corresponding values of them are 0.941 W and 0.777 W respectively. The maximum energy efficiency of the system is 0.796. There is an interesting result existing here. Because, power output increases with decreasing  $\epsilon$  value, only way of this is that  $\epsilon$  should have an optimum point and this value is equal to 0.82.

In Figs. 4 and 5, it can be seen that variation of the considered parameters with  $\omega_b$  ( $\epsilon_H = \epsilon_L = 0.9$ ) and ( $\epsilon_H = \epsilon_L = 0.8$ ). For  $\epsilon_H = \epsilon_L = 0.9$ , power output and ecological function have an optimum point. The optimum point of the power output is obtained at 2.7 with 0.934 W and the optimum point of the ecological function is obtained at 2 with 0.557 W. Energy efficiency decreases linearly and its maximum is 0.780. For  $\epsilon_H = \epsilon_L = 0.8$ , parameters have same tendency. The maximum power output and ecological function are obtained at 2.6 and 2 respectively and corresponding values are 1.064 W and 0.637 W. The energy efficiency continuously decreases and its maximum is 0.779. Special case mentioned above is valid for this case too.

In Fig. 6, one can see P-η-E curve for  $\omega_a$  ( $\epsilon_H = \epsilon_L = 0.9$ ) and

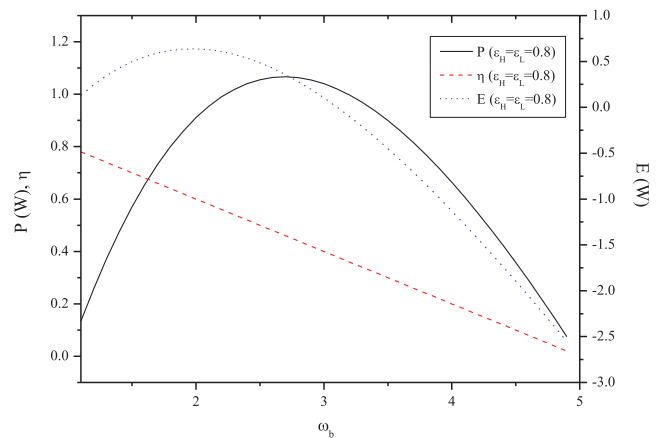


Fig. 5. Change of P, η and E with  $\omega_b$  for  $\epsilon_H = \epsilon_L = 0.8$ .

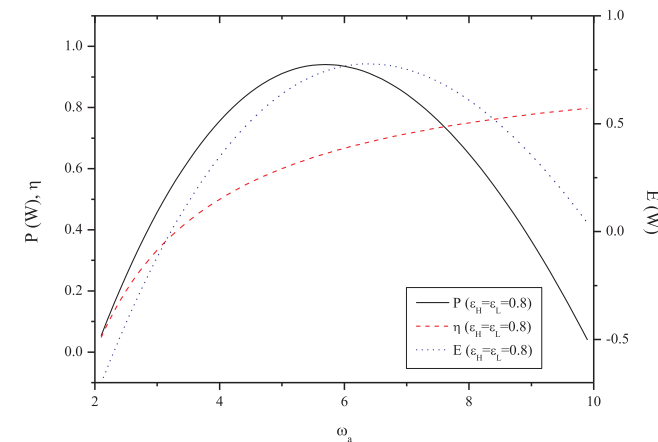


Fig. 3. Change of P, η and E with  $\omega_a$  for  $\epsilon_H = \epsilon_L = 0.8$ .

( $\epsilon_H = \epsilon_L = 0.8$ ). For ( $\epsilon_H = \epsilon_L = 0.9$ ), ecological function at maximum power is 0.650 W which is 96% of the maximum ecological function and power output at maximum ecological function is equal to 0.797 W and it is 97% of maximum power. Similarly, power at the maximum efficiency is 0.035 W is 4% of maximum power and efficiency at maximum power is 0.650 which is 81% of the maximum efficiency. The efficiency at the maximum ecological function is equal to 0.687 and it is 86% of its maximum. Finally, ecological function at the maximum efficiency 0.034 and it is 5% of maximum ecological function and similar results are true for  $\epsilon_H = \epsilon_L = 0.8$  conditions

In Fig. 7, one can see P-η-E curve for  $\omega_b$  ( $\epsilon_H = \epsilon_L = 0.9$ ) and ( $\epsilon_H = \epsilon_L = 0.8$ ). For ( $\epsilon_H = \epsilon_L = 0.9$ ), ecological function at maximum power is 0.313 W which is 61% of the maximum ecological function and power output at maximum ecological function is equal to 0.796 W and it is 85% of maximum power. Similarly, power at the maximum efficiency is 0.117 W and it is 13% of maximum power and efficiency at maximum power is 0.460 which is 59% of the maximum efficiency. The efficiency at the maximum ecological function is equal to 0.600 and it is 77% of its maximum. Finally, ecological function at the maximum efficiency 0.114 and it is 20% of maximum ecological function and similar results are valid for  $\epsilon_H = \epsilon_L = 0.8$  conditions.

According to these results, all optimum points for the system are defined. In addition, it can be seen that optimum ecological function should be selected for the best operating conditions. Because under these conditions, power output and energy efficiency output have bigger points and in this point, power and exergy destruction difference is the maximum. Finally, under optimum ecological function, system can be operated under most appropriate conditions.

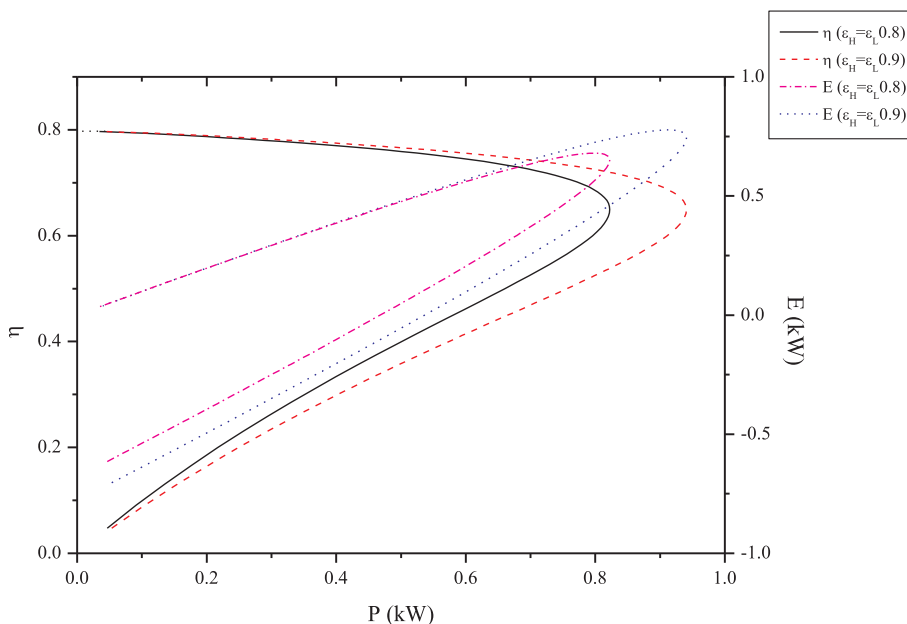


Fig. 6. P-η-E curve with  $\omega_a$ .

4. Conclusions

In this paper, an irreversible quantum heat engine is researched in terms of power output, energy efficiency and exergetic sustainability index. It is tried to obtain most convenient operation conditions.

It is determined that power output and ecological function have optimum points and these points should be considered when a quantum engine is designed. In addition, operating cycle at the ecological function optimum point provides most convenient conditions for the cycle. Besides, ecological function obtained a balance between power and exergy destruction. Finally, it is recommended that ecological function should be considered on the quantum heat engine designing.

Conflict of interest

We wish to confirm that there are no known conflicts of interest associated with this publication and there has been no significant financial support for this work that could have influenced its outcome.

Acknowledgement

Authors would like to thank the reviewers for their valuable comments, which have been utilized in improving the quality of the paper.

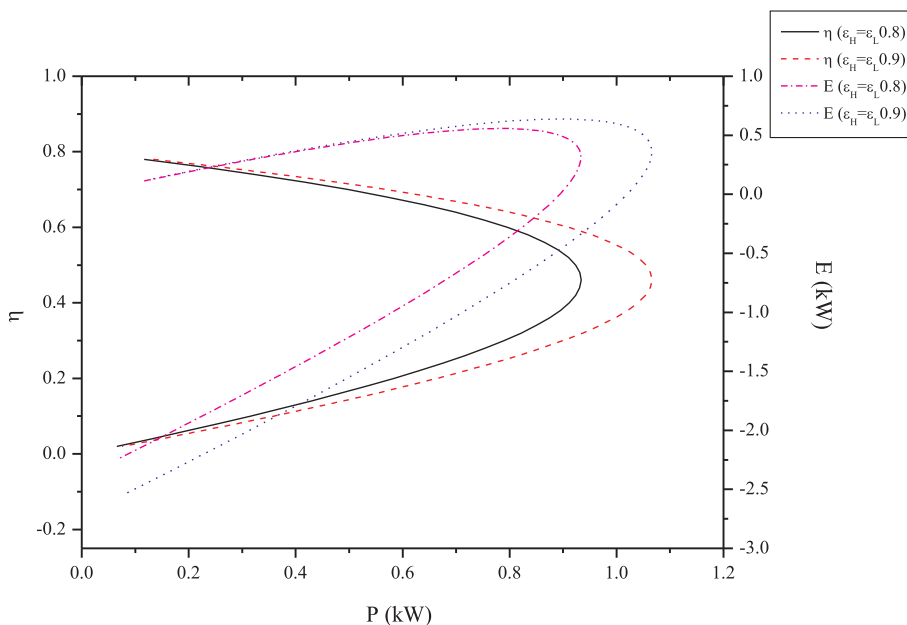


Fig. 7. P-η-E curve with  $\omega_b$ .

## References

- [1] F.L. Curzon, B. Ahlborn, Efficiency of a Carnot engine at maximum power-output, *Am. J. Phys.* 43 (1975) 22–24.
- [2] I.I. Novikov, The efficiency of atomic power stations, *J. Nucl. Energy* 11 (1958) 25–28.
- [3] F. Wu, L. Chen, F. Sun, C. Wu, F. Guo, Q. Li, Quantum degeneracy effect on performance of irreversible Otto cycle with ideal Bose gas, *Energy Convers. Manage.* 47 (2006) 3008–3018.
- [4] W. Nie, Q. Liao, C.Q. Zhang, J. He, Micro-/nanoscaled irreversible Otto engine cycle with friction loss and boundary effects and its performance characteristics, *Energy* 35 (2010) 4658–4662.
- [5] J. Guo, X. Zhang, G. Su, J. Chen, The performance analysis of a micro-/nanoscaled quantum heat engine, *Phys. A* 391 (2012) 6432–6439.
- [6] E. Açıkkalp, N. Caner, Determining performance of an irreversible nano scale dual cycle operating with Maxwell-Boltzmann gas, *Phys. A* 424 (2015) 342–349.
- [7] F. Wu, L. Chen, S. Wu, Ecological optimization performance of an irreversible quantum Otto cycle working with an ideal Fermi gas, *Open Syst. Inf. Dyn.* 13 (2006) 55–66.
- [8] E. Açıkkalp, N. Caner, Determining of the optimum performance of a nano scale irreversible Dual cycle with quantum gases as working fluid by using different methods, *Phys. A* 433 (2015) 247–258.
- [9] E. Açıkkalp, N. Caner, Performance of an irreversible nano Brayton cycle operating with Maxwell-Boltzmann Gas, *Eur. Phys. J. Plus* 130 (2015) 93.
- [10] H. Wang, S. Liu, J. He, Performance analysis and parametric optimum criteria of a quantum Otto heat engine with heat transfer effects, *Appl. Therm. Eng.* 29 (2009) 706–711.
- [11] A. Şişman, H. Saygin, On the power cycles working with ideal quantum gases: I. The Ericsson cycle, *J. Phys. D: Appl. Phys.* 32 (1999) 664–670.
- [12] J. Chen, J. He, B. Hua, The influence of regenerative losses on the performance of a Fermi Ericsson refrigeration cycle, *J. Phys. A: Math. Gen.* 35 (2002) 7995–8004.
- [13] A. Şişman, H. Saygin, Efficiency analysis of a stirling power cycle under quantum degeneracy conditions, *Phys. Scr.* 63 (2001) 263–267.
- [14] A. Şişman, H. Saygin, Re-optimization of Otto power cycles working with ideal quantum gasses, *Phys. Scr.* 64 (2001) 108–112.
- [15] H. Wang, S. Liu, J. Du, Performance analysis and parametric optimum criteria of a regeneration Bose-Otto engine, *Phys. Scr.* 79 (2009) 055004.
- [16] W. Hao, W. Guo-Xing, Optimization criteria of a Bose Brayton heat engine, *Chin. Phys. B* 21 (2012) 010505.
- [17] A. Şişman, H. Saygin, The improvement effect of quantum degeneracy on the work from a Carnot cycle, *Appl. Energy* 68 (2001) 367–376.
- [18] B. Lin, J. Chen, The influence of quantum degeneracy on the performance of a Fermi Brayton engine, *Open Sys. Inform. Dyn.* 11 (2004) 87–99.
- [19] F. Wu, L. Chen, S. Wu, Optimization criteria for an irreversible quantum Brayton engine with an ideal Bose gas, *J. Appl. Phys.* 99 (2006) 54904–54909.
- [20] X. Liu, L. Chen, F. Wu, F. Sun, Optimal performance of a spin quantum Carnot heat engine with multi-irreversibilities, *J. Energy Inst.* 87 (2014) 69–80.
- [21] L. Chen, X. Liu, Y. Ge, F. Wu, F.I. Sun, Ecological optimization of an irreversible harmonic oscillators Carnot refrigerator, *J. Energy Inst.* 86 (2013) 85–96.
- [22] X. Liu, L. Chen, Y. Ge, F. Wu, F. Sun, Fundamental optimal relation of a spin 1/2 quantum Brayton heat engine with multi-irreversibilities, *Sci. Iranica Trans. B Mech. Eng.* 19 (2012) 1124–1132.
- [23] Liu Xiaowei, Chen Lingen, Wu Feng, Sun Fengrui, Ecological optimization of an irreversible quantum Carnot heat engine with spin-1/2 systems, *Phys. Scr.* 81 (2010) 025003.
- [24] J. Chen, B. Lin, B. Hua, The Performance Analysis of a Quantum Heat Engine Working with Spin Systems, *J. Phys. D: Appl. Phys.* 35 (2002) 2051–2065.
- [25] B. Lin, J. Chen, Optimal analysis of the performance of an irreversible quantum heat engine with spin systems, *J. Phys. A: Math. Gen.* 38 (2005) 69–79.
- [26] J. Wang, J. He, Y. Xin, Performance analysis of a spin quantum heat Engine cycle with internal friction, *Phys. Scr.* 75 (2007) 227–234.
- [27] E. Geva, R. Kosloff, A quantum-mechanical heat engine operating in finite time a model consisting of spin-1/2 systems as working fluid, *J. Chem. Phys.* 96 (1992) 3054–3067.
- [28] J.C. Chen, B.H. Lin, B. Hua, The performance of a quantum heat engine working with spin systems, *J. Phys. D: Appl. Phys.* 35 (2002) 2051–2057.
- [29] F. Wu, L.G. Chen, F.R. Sun, C. Wu, Performance of an irreversible quantum Carnot engine with spin-1/2, *J. Chem. Phys.* 124 (2006) 214702.
- [30] F. Wu, L.G. Chen, F.R. Sun, C. Wu, Q. Li, Generalized model and optimum performance of an irreversible quantum Brayton engine with spin systems, *Phys. Rev. E* 7 (2006) 016103.
- [31] G.P. Beretta, Quantum thermodynamic Carnot and Otto-like cycles for a two-level system, *EPL* 99 (2012) 20005.
- [32] M.J. Henrich, F. Rempp, G. Mahler, Quantum thermodynamic Otto machines: a spin-system approach, *Eur. Phys. J. Special Topics* 151 (2007) 157–165.
- [33] M. Azimi, L. Chotorlishvili, S.K. Mishra, T. Vekua, W. Hübner, J. Berakdar, Quantum Otto heat engine based on a multiferroic chain working substance, *New J. Phys.* 16 (2014) 063018.
- [34] W. Hübner, G. Lefkidis, C.D. Dong, D. Chaudhuri, Spin-dependent Otto quantum heat-engine based on molecular substance, *Phys. Rev. B* 90 (2014) 024401.
- [35] H. Wang, G. Wu, D. Chen, Thermal entangled quantum Otto engine based on the two qubits Heisenberg model with Dzyaloshinskii–Moriya interaction in an external magnetic field, *Phys. Scr.* 86 (2012) 015001 (8 pp).
- [36] M.H. Ahmadi, M.A. Ahmadi, F. Pourfayaz, M. Bidi, Entransy analysis and optimization of performance of nano-scale irreversible Otto cycle operating with Maxwell-Boltzmann ideal gas, *Chem. Phys. Lett.* 658 (2016) 293–302.
- [37] M.H. Ahmadi, M.A. Ahmadi, F. Pourfayaz, Performance assessment and optimization of an irreversible nano-scale Stirling engine cycle operating with Maxwell-Boltzmann gas, *Eur. Phys. J. Plus* 130 (9) (2015) 1–13.
- [38] S.A. Sadatsakkak, M.H. Ahmadi, M.A. Ahmadi, Optimization performance and thermodynamic analysis of an irreversible nano scale Brayton cycle operating with Maxwell-Boltzmann gas, *Energy Convers. Manage.* 101 (2015) 592–605.
- [39] A. Dalkiran, E. Açıkkalp, N. Caner, Analysis of a quantum irreversible Otto cycle with exergetic sustainable index, *Phys. A* 453 (2016) 316–326.
- [40] E. Açıkkalp, N. Caner, Application of exergetic sustainability index to a nano-scale irreversible Brayton cycle operating with ideal Bose and Fermi gasses, *Phys. Lett. A* 379 (2015) 1990–1997.
- [41] J. He, H. Wang, S. Liu, Performance characteristics of a quantum Diesel refrigeration cycle, *Energy Convers. Manage.* 50 (2009) 933–937.
- [42] H. Saygin, A. Şişman, Brayton refrigeration cycles working under quantum degeneracy conditions, *Appl. Energy* 69 (2001) 77–85.
- [43] J. He, J. Chena, B. Hua, Influence of quantum degeneracy on the performance of a Stirling refrigerator working with an ideal Fermi gas, *Appl. Energy* 72 (2002) 541–554.
- [44] Y. Yang, B. Lin, J. Chen, Influence of regeneration on the performance of a Brayton refrigeration-cycle working with an ideal Bose-gas, *Appl. Energy* 83 (2006) 99–112.
- [45] B. Lin, Y. Zhang, J. Chen, Influence of quantum degeneracy and regeneration on the performance of Bose-Stirling refrigeration cycles operated in different temperature regions, *Appl. Energy* 83 (2006) 513–535.
- [46] H. Wang, S. Liu, J. He, Optimum criteria of an irreversible quantum Brayton refrigeration cycle with an ideal Bose gas, *Phys. B* 403 (2008) 3867–3878.
- [47] J. Liu, B. Lin, W. Hu, J. Chen, Influence of multi-irreversibilities on the performance of a Brayton refrigeration cycle working with an ideal Bose or Fermi gas, *Int. J. Therm. Sci.* 47 (2008) 1374–1381.
- [48] Y. Zhang, B. Lin, Chen J., The influence of quantum degeneracy and irreversibility on the performance of a Fermi quantum refrigeration cycle, *J. Phys. A: Math. Gen.* 37 (2004) 7485–7497.
- [49] B. Lin, J. Chen, The performance Analysis of a Quantum Brayton Refrigeration cycle with ideal Bose gas, *Open Sys. Inf. Dyn.* 10 (2003) 147–157.
- [50] F. Wu, L. Chen, F. Sun, C. Wu, F. Guo, Optimal performance of an irreversible quantum Brayton refrigerator with ideal Bose gases, *Phys. Scr.* 73 (2006) 452–457.
- [51] E. Açıkkalp, N. Caner, Application of exergetic sustainable index to the quantum irreversible Diesel refrigerator cycles for 1-D box system, *Eur. Phys. J. Plus* 130 (2015) 73.
- [52] X. Liu, L. Chen, F. Wu, F. Sun, Fundamental optimal relation of an irreversible quantum Carnot heat pump with spin-1/2 systems, *Math. Comput. Model.* 54 (2011) 190–202.
- [53] X. Liu, L. Chen, F. Wu, F. Sun, Cooling load and energy efficiency optimization of an irreversible Carnot refrigerator with spin-1/2 systems, *Int. J. Energy Environ.* 2 (2011) 797–812.
- [54] T. Feldmann, R. Kosloff, Performance of discrete heat engines and heat pumps in finite time, *Phys. Rev. E* 61 (2000) 4774–4790.
- [55] J. He, Y. Xin, X. He, Performance optimization of quantum Brayton refrigeration cycle working with spin system, *Appl. Energy* 84 (2007) 176–186.
- [56] B.H. Lin, J.C. Chen, Performance analysis of a quantum heat-pump using spin systems as the working substance, *Appl. Energy* 78 (2004) 75–93.
- [57] J.Z. He, J.C. Chen, B. Hua, Quantum refrigeration cycles using spin-1/2 systems as the working substance, *Phys. Rev. E* 65 (2002) 036145.
- [58] Xiao-Li Huang, Xin-Ya Niu, Xiao-Ming Xiu, Xue-Xi Yi, Quantum Stirling heat engine and refrigerator with single and coupled spin systems, *Eur. Phys. J. D* 68 (2014) 32.
- [59] M.H. Ahmadi, M.A. Ahmadi, A. Maleki, F. Pourfayaz, M. Bidi, E. Açıkkalp, Exergetic sustainability evaluation and multi-objective optimization of performance of an irreversible nano scale Stirling refrigeration cycle operating with Maxwell-Boltzmann gas, *Renewable Sustainable Energy Rev.* 78 (2017) 80–92.
- [60] M.A. Ahmadi, M.A. Nabakhteh, M.A. Ahmadi, F. Pourfayaz, M. Bidi, Investigation and optimization of performance of nano-scale Stirling refrigerator using working fluid as Maxwell-Boltzmann gases, *Phys. A: Stat. Mech. Appl.* 483 (2017) 337–350.
- [61] A. Dalkiran, E. Açıkkalp, A.F. Savas, Analysis of a nano-scale thermo-acoustic refrigerator, *Int. J. Refrig.* 66 (2016) 1–9.
- [62] F. Wu, L.G. Chen, F.R. Sun, C. Wu, P.Q. Hua, Optimum performance parameters for a quantum Carnot heat pump with spin-1/2, *Energy Convers. Manage.* 39 (11) (1998) 1161–1167.
- [63] F. Wu, L.G. Chen, F.R. Sun, Exergetic efficiency optimization for an irreversible quantum Brayton refrigerator with spin systems, *Appl. Math. Model.* 34 (2010) 617–625.
- [64] F. Angulo-Brown, An ecological optimization criterion for finite-time heat engines, *J. Appl. Phys.* 69 (1991) 7465–7469.
- [65] Z. Yan, Comment on ecological optimization criterion for finite-time heat-engines, *J. Appl. Phys.* 73 (1993) 3583.