

# Countable and separable elementary soft topological space

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This paper is an introduction to countable and separable elementary soft topological spaces, which includes concepts such as dense soft set, first countability, second countability, separability and Lindelöf properties and some basic properties of them in the elementary soft topological spaces.

## KEYWORDS

countable soft space, dense soft set, separable soft space, soft element, soft set

## MSC CLASSIFICATION

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## 1 | INTRODUCTION

Molodtsov<sup>1</sup> introduced the soft sets theory, in 1999. He applied this theory to solve some practical problems in medical science, economics, social science, etc. After these, Maji et al<sup>2,3</sup> examined the soft set theory a great length and also applied soft sets to decision making problems. Some researchers<sup>4-17</sup> have established different topologies on the soft sets and have viewed their topological properties. Das and Samanta<sup>18,19</sup> gave the concepts such as soft elements, soft real numbers and soft complex numbers. Samanta et al. and several authors studied the soft metric, soft vector, soft norm, etc. concepts via soft elements.<sup>20-25</sup> Taşköprü and Altıntaş<sup>26</sup> established a new topological structure on the soft sets using elementary operations ( $\varepsilon$ -operations), which called elementary soft topology ( $\varepsilon$ -soft topology). In recent years, many mathematical structures have been established on the soft sets and their applications have been given to different field of science.<sup>5,16,27-43</sup>

In this paper, we examine the concept such as dense soft set, countability, separability and Lindelöf's theorems in  $\varepsilon$ -soft topological spaces. The paper is planned as below: Section 2 contains some basic information about the soft set theory to be used in other sections. In Section 3, we define first countable  $\varepsilon$ -soft space and give some examples. We prove that the first countable  $\varepsilon$ -soft topological space has the hereditary property and topological property. Then, we define second countable  $\varepsilon$ -soft space and give some examples. We prove that the property of being a second countable  $\varepsilon$ -soft space is both a hereditary property and a topological property. We show that every second countable  $\varepsilon$ -soft space is a first countable  $\varepsilon$ -soft space. In this section, we also define soft cover and prove that every soft open cover of a soft set in a second countable  $\varepsilon$ -soft topological space has a countable soft open subcover. In Section 4, we first define the concept of dense soft set containing the soft elements in  $\varepsilon$ -soft topological spaces and give some examples. In this section, we also define separable  $\varepsilon$ -soft topological space and give basic properties of it. We prove that every second countable  $\varepsilon$ -soft space is a separable space. Also, we research Lindelöf properties of  $\varepsilon$ -soft topological spaces.

## 2 | PRELIMINARIES

**Definition 1**<sup>1</sup>. Assume that  $P$  is a parameters set,  $X$  is a non-empty set and  $P(X)$  is the power set of  $X$ . We call a pair  $(G, P)$  a soft set on  $X$  if  $G : P \rightarrow P(X)$  is a mapping. That is, a soft set on  $X$  is a parametrized class of subsets in  $X$ . We can regard a soft set as the  $\alpha$ -approximate elements set of  $(G, P)$  for every  $\alpha \in P$ .

We call a function  $\varepsilon : P \rightarrow X$  a soft element of  $X$ . If for each  $\alpha \in P$ ,  $\varepsilon(\alpha) \in G(\alpha)$ ,  $\varepsilon$  belongs to  $(G, P)$ . If  $G(\alpha)$  is a singleton set for each  $\alpha \in P$ , then the soft set  $(G, P)$  itself can be taken as a soft element. The soft elements are denoted  $\tilde{r}$ ,  $\tilde{s}$ ,  $\tilde{t}$ , etc. and the class of soft elements of  $(G, P)$  by  $SE(G, P)$ . For details, see<sup>18</sup>.

**Definition 2**<sup>18</sup>. Assume that  $P$  is a parameters set and  $B(\mathbb{R})$  is the class of nonempty all bounded subsets of real numbers set  $\mathbb{R}$ . We call a mapping  $G : P \rightarrow B(\mathbb{R})$  a soft real set and denote by  $(G, P)$ . If  $G(\alpha)$  is a singleton set for each  $\alpha \in P$ , then  $(G, P)$  is a soft real number. Soft real numbers are denoted  $\tilde{s}$ ,  $\tilde{t}$ , etc. Specially, for each  $\alpha \in P$ , a soft real number satisfying  $\tilde{s}(\alpha) = s$ , is denoted by  $\bar{s}$ . We denote all the soft real numbers class and non-negative soft real numbers class with  $\mathbb{R}(P)$  and  $\mathbb{R}(P)^*$ , respectively. In addition, for,

$$\begin{aligned} \tilde{r} \lesssim \tilde{s} & \text{ if } \tilde{r}(\alpha) \leq \tilde{s}(\alpha), \forall \alpha \in P, & \tilde{r} \gtrsim \tilde{s} & \text{ if } \tilde{r}(\alpha) \geq \tilde{s}(\alpha), \forall \alpha \in P, \\ \tilde{r} \lessdot \tilde{s} & \text{ if } \tilde{r}(\alpha) < \tilde{s}(\alpha), \forall \alpha \in P, & \tilde{r} \gtrdot \tilde{s} & \text{ if } \tilde{r}(\alpha) > \tilde{s}(\alpha), \forall \alpha \in P. \end{aligned}$$

**Definition 3.** Assume that  $(G, P)$  and  $(H, P)$  are soft sets on  $X$ . We call  $(G, P)$  a null soft set if  $G(\alpha) = \emptyset$  and call  $(G, P)$  an absolute soft set if  $G(\alpha) = X$  for each  $\alpha \in P$ , denote them by  $\Phi$  and  $\tilde{X}$ , respectively.

The class of  $\Phi$  and the soft sets  $(G, P)$  on  $X$  such that  $G(\alpha) \neq \emptyset$  for every  $\alpha \in P$  denoted by  $S(\tilde{X})$ .

Each class  $\mathcal{B}$  of soft elements of a soft set can generate a soft subset of this soft set. The soft set generated with  $\mathcal{B}$  is denoted by  $SS(\mathcal{B})$ . For any soft set  $(G, P) \in S(\tilde{X})$ ,  $SS(SE(G, P)) = (G, P)$ ; whereas for a class  $\mathcal{B}$  of soft elements,  $SE(SS(\mathcal{B})) \supset \mathcal{B}$ .

We call  $(G, P)$  a soft subset of  $(H, P)$  if  $G(\alpha) \subset H(\alpha)$  for each  $\alpha \in P$  and denote by  $(G, P) \check{C}(H, P)$ . Then, we call  $(H, P)$  a soft upper set of  $(G, P)$  and denote by  $(H, P) \check{S}(G, P)$ .  $(G, P) = (H, P)$  if and only if  $(G, P) \check{C}(H, P)$  and  $(H, P) \check{S}(G, P)$ .

The union  $(F, P)$  of  $(H, P)$  and  $(G, P)$ , denoted by  $(F, P) = (H, P) \check{\cup}(G, P)$ , is defined as  $F(\alpha) = H(\alpha) \cup G(\alpha)$ . The intersection  $(F, P)$  of  $(H, P)$  and  $(G, P)$ , denoted by  $(F, P) = (H, P) \check{\cap}(G, P)$ , is defined as  $F(\alpha) = H(\alpha) \cap G(\alpha)$ . The difference  $(F, P)$  of  $(H, P)$  and  $(G, P)$ , denoted by  $(F, P) = (H, P) \check{\setminus}(G, P)$ , is defined as  $F(\alpha) = H(\alpha) \setminus G(\alpha)$ . The complement  $(G, P)^c = (G^c, P)$  of  $(G, P)$  is defined as a mapping  $G^c : P \rightarrow P(X)$  given by  $G^c(\alpha) = X \setminus F(\alpha)$ ,  $\forall \alpha \in P$ .

The elementary union  $(F, P) = (H, P) \check{\cup} (G, P)$  and elementary intersection  $(F, P) = (H, P) \check{\cap} (G, P)$  of  $(H, P)$ ,  $(G, P) \in S(\tilde{X})$  are defined by  $(F, P) = SS(SE(H, P) \cup SE(G, P))$  and  $(F, P) = SS(SE(H, P) \cap SE(G, P))$ , respectively. (For details, see<sup>26</sup>).

Throughout the work, we also use  $G$  briefly instead of  $(G, P)$

**Definition 4**<sup>26</sup>. Assume that  $X, Y$  are two non-empty sets and  $P$  is a parameters set. We call a mapping  $f : SE(\tilde{X}) \rightarrow SE(\tilde{Y})$  a soft mapping and call it a soft function if  $\{f(\tilde{r})(\alpha) : \tilde{r}(\alpha) = a, \tilde{r} \in SE(\tilde{X}), \alpha \in P, a \in X\}$  is a singleton set.

The soft mapping  $f$  is a soft surjective mapping, if  $f(SE(\tilde{X})) = SE(\tilde{Y})$  and  $f$  is soft bijective mapping if  $f(\tilde{x}) \neq f(\tilde{y}) \in SE(\tilde{Y})$  when  $\tilde{x} \neq \tilde{y}$  for each  $\tilde{x}, \tilde{y} \in SE(\tilde{X})$ .

**Definition 5**<sup>20</sup>. Assume that  $P$  is a parameters set and  $X$  is a non-empty set. We call a mapping  $d : SE(\tilde{X}) \times SE(\tilde{X}) \rightarrow \mathbb{R}(P)^*$  that meets the following condition a soft metric on  $\tilde{X}$  and call the triple  $(\tilde{X}, d, P)$  a soft metric space. Let  $\tilde{r}, \tilde{s}, \tilde{t} \in SE(\tilde{X})$ :

- M1.  $d(\tilde{r}, \tilde{s}) \gtrsim \bar{0}$ ,
- M2.  $\tilde{r} = \tilde{s}$  if and only if  $d(\tilde{r}, \tilde{s}) = \bar{0}$ ,
- M3.  $d(\tilde{r}, \tilde{s}) = d(\tilde{s}, \tilde{r})$ ,
- M4.  $d(\tilde{r}, \tilde{t}) \lesssim d(\tilde{r}, \tilde{s}) + d(\tilde{s}, \tilde{t})$ .

**Theorem 1**<sup>21</sup>. Let  $X$  be non-empty universal set and  $P$  be non-empty set of parameters. If a soft metric  $d$  over  $\tilde{X}$  meets the following axiom M5,  $d_\alpha : X \times X \rightarrow \mathbb{R}^+$  which is defined with  $d_\alpha(\tilde{x}(\alpha), \tilde{y}(\alpha)) = d(\tilde{x}, \tilde{y})(\alpha)$  for each  $\tilde{x}, \tilde{y} \in \tilde{X}$  is a metric over  $X$  for every  $\alpha \in P$ .

- M5.  $\{d(\tilde{x}, \tilde{y})(\alpha) : \tilde{x}(\alpha) = \eta, \tilde{y}(\alpha) = \xi\}$  is one-element set for each  $\alpha \in P$ , each  $\tilde{x}, \tilde{y} \in \tilde{X}$  and each  $(\eta, \xi) \in X \times X$ .

**Definition 6**<sup>26</sup>. We call a soft sets class  $\mathcal{T} \subset S(\tilde{X})$  a elementary soft topology if it meets the following axioms:

1.  $\Phi, \tilde{X} \in \mathcal{T}$ ,
2. The finite elementary intersection of soft sets of  $\mathcal{T}$  belongs to  $\mathcal{T}$ ,
3. The arbitrary elementary union of soft sets of  $\mathcal{T}$  belongs to  $\mathcal{T}$ .

Then we call the triple  $(X, \mathcal{T}, P)$  an  $\varepsilon$ -soft topological space.

We call each set of the class  $\mathcal{T}$  a soft open set and call a soft set  $K$  a soft closed set if  $K \in S(\tilde{X})$  and  $K^c \in \mathcal{T}$ .

We call a soft set  $N \in S(\tilde{X})$  is called soft neighborhood of the soft element  $\tilde{x}$  if there exists a soft open set  $U \in \mathcal{T}$  with  $\tilde{x} \in U \subseteq N$ . We denote the class of all soft neighborhoods of the soft element  $\tilde{x}$  by  $N(\tilde{x})$ .

We call a soft element  $\tilde{x}$  of a soft set  $G \in S(\tilde{X})$  a soft interior element of the soft set  $G$  if there is  $U \in \mathcal{T}$  with  $\tilde{x} \in U \subseteq G$  and denote the class of all soft interior elements of the soft set  $G$  by  $intG$ . A soft set which produced by  $intG$  is called soft interior of  $G$  and is denoted by  $G^\circ = SS(intG)$ .

We call a soft element  $\tilde{x} \in \tilde{X}$  a soft closure element of the soft set  $G \in S(\tilde{X})$  if for every  $N \in N_{\mathcal{T}}(\tilde{x})$ ,  $N \cap G \neq \Phi$  and denote the class of all soft closure elements of  $G$  by  $clG$ . A soft set which produced by  $clG$  is called soft closure of  $G$  and it is denoted by  $\bar{G} = SS(clG)$ .

We call a soft element  $\tilde{x} \in \tilde{X}$  a soft limit element of a soft set  $G \in S(\tilde{X})$  if for every  $N \in N(\tilde{x})$ ,  $G \cap (N \setminus \tilde{x}) \neq \Phi$  and denote the class of all soft limit elements of  $G$  by  $limF$ . A soft set which produced by  $limG$  is called soft limit set of  $G$  and it is denoted by  $G' = SS(limG)$ .

We call a soft sets class  $\mathcal{B} \subset S(\tilde{X})$  a soft base for any elementary soft topology over  $\tilde{X}$  if (1) there is a soft set  $B \in \mathcal{B}$  with  $\tilde{r} \in B$  for each soft element  $\tilde{r} \in \tilde{X}$ , (2) if for  $B_1, B_2 \in \mathcal{B}$ ,  $\tilde{r} \in B_1 \cap B_2$ , there is a soft set  $B_3$  with  $\tilde{r} \in B_3 \subseteq B_1 \cap B_2$ .

We call a soft open sets class  $\mathcal{B}_{\tilde{x}}$  which contains a soft element  $\tilde{x} \in \tilde{X}$  a soft local base of the soft element  $\tilde{x}$  if there is  $B_{\tilde{x}} \in \mathcal{B}_{\tilde{x}}$  with  $\tilde{x} \in B_{\tilde{x}} \subseteq U$  for every soft set  $U$  that contains the soft element  $\tilde{x}$ .

**Example 1**<sup>26</sup>. Every  $\varepsilon$ -soft metric spaces  $(\tilde{X}, d, P)$  that meets the the axiom M5 is a  $\varepsilon$ -soft topological space.

**Definition 7**<sup>44</sup>. Assume that  $(\tilde{X}, \mathcal{T}, P)$  is an  $\varepsilon$ -soft topological space,  $F$  is a soft set in  $S(\tilde{X})$ . A class  $\mathcal{T}_F = \{F \cap U : U \in \mathcal{T}\}$  is an  $\varepsilon$ -soft topology over  $F$  if  $F \cap U \in S(\tilde{X})$  for every  $U \in \mathcal{T}$ . We call the  $\varepsilon$ -soft topology  $\mathcal{T}_F$  which is reduced to over the soft set  $F$  from the topology  $\mathcal{T}$  a  $\varepsilon$ -soft subspace topology and call the space  $(F, \mathcal{T}_F, P)$   $\varepsilon$ -soft subspace of the space  $(\tilde{X}, \mathcal{T}, P)$ .

**Definition 8**<sup>26</sup>. Assume that  $(\tilde{X}, \mathcal{T}, P)$  and  $(\tilde{Y}, \mathcal{T}', P)$  are two  $\varepsilon$ -soft topological space,  $\tilde{x} \in \tilde{X}$  and  $f : SE(\tilde{X}) \rightarrow SE(\tilde{Y})$  is a soft mapping. We call  $f$  soft continuous at  $\tilde{x}$  if there is at least one soft neighborhood  $N \in N_{\mathcal{T}}(\tilde{x})$  satisfying  $f(SE(N)) \subseteq SE(N')$  for every soft neighborhood  $N' \in N_{\mathcal{T}'}(f(\tilde{x}))$ .

We call  $f$  a soft open mapping if  $SS(f(SE(U))) = G \in S(\tilde{Y})$  is a soft open set for each  $U \in \mathcal{T}$  and call  $f$  a soft closed mapping if  $SS(f(SE(F))) = H \in S(\tilde{Y})$  is a soft closed set for each closed set  $F \in S(\tilde{X})$ .

We call  $f$  a soft homeomorphism if it is bijective, surjective, soft continuous and  $f^{-1}$  is soft continuous.

**Remark 1.** Assume that  $(\tilde{X}, \mathcal{T}, P)$  and  $(\tilde{Y}, \mathcal{T}', P)$  are two  $\varepsilon$ -soft topological spaces and  $f : SE(\tilde{X}) \rightarrow SE(\tilde{Y})$  is a soft mapping. The following statements are equivalent.

1.  $f$  is a soft homeomorphism.
2.  $f$  is a bijective, surjective, continuous and soft open function.
3.  $f$  is a bijective, surjective, continuous and soft closed function.

**Definition 9.** A property of an  $\varepsilon$ -soft topological space  $(\tilde{X}, \mathcal{T}, P)$  is said to be hereditary iff every soft subspace of  $(\tilde{X}, \mathcal{T}, P)$  also possesses that property.

A property of an  $\varepsilon$ -soft topological space  $(\tilde{X}, \mathcal{T}, P)$  is said to be topological property iff every soft space soft homeomorphic to  $(\tilde{X}, \mathcal{T}, P)$  also possess that property.

### 3 | SOFT COUNTABILITY

#### 3.1 | First countable soft spaces

**Definition 10.** We call an  $\varepsilon$ -soft topological space  $(\tilde{X}, \mathcal{T}, P)$  a first countable if there exists a countable soft local base at every soft element  $\tilde{x} \in \tilde{X}$ .

**Example 2.** Let  $P$  be finite parameters set. The soft real topological space  $(\tilde{\mathbb{R}}, \mathcal{T}, A)$  is a first countable  $\varepsilon$ -soft space. In fact, for every soft element  $\tilde{x} \in \mathbb{R}(A)$ , the class soft open interval  $B(\tilde{x}) = \left\{ \left( \tilde{x} - \frac{1}{n}, \tilde{x} + \frac{1}{n} \right) : \frac{1}{n}(\alpha) = \frac{1}{n} \in \mathbb{R}, \forall \alpha \in P \right\}$  is a countable local base at soft element  $\tilde{x}$ .

**Example 3.** All  $\varepsilon$ -soft metric spaces  $(\tilde{X}, d, P)$  that meets the axiom M5,  $(\tilde{X}, d, P)$  is a first countable  $\varepsilon$ -soft space.

**Example 4.** The  $\varepsilon$ -soft discrete space is a first countable space. In fact, for every class  $B(\tilde{x}) = \{ \{\tilde{x}\} : \tilde{x} \in \tilde{X} \}$  is a soft open set and is contained in every soft open set  $U$  containing soft element  $\tilde{x}$ . Hence  $B(\tilde{x})$  is a soft local neighborhood and countable.

**Theorem 2.** Assume that  $(\tilde{X}, \mathcal{T}, P)$  is a first countable  $\varepsilon$ -soft topological space and  $F \in S(\tilde{X})$  is a soft set. Then the  $\varepsilon$ -soft subspace  $(F, \mathcal{T}_F, P)$  is first countable.

*Proof.* Let us taken an element  $\tilde{x}$  of  $F$ . Since  $F \in S(\tilde{X})$ ,  $\tilde{x} \in \tilde{X}$ . Then, there is a countable soft local base at  $\tilde{x} \in \tilde{X}$  according to the topology  $\mathcal{T}$  since  $(\tilde{X}, \mathcal{T}, P)$  is a first countable  $\varepsilon$ -soft space. So,  $B_F(\tilde{x}) = \{F \cap B_n : n \in \mathbb{N}, F \cap B_n \in S(\tilde{X})\}$  is a countable soft local base at the element  $\tilde{x}$  according to the topology  $\mathcal{T}_F$ . Thus,  $(F, \mathcal{T}_F, P)$  is a first countable  $\varepsilon$ -soft space.  $\square$

**Theorem 3.** The property of being a first countable is a topological property.

*Proof.* Assume that  $(\tilde{X}, \mathcal{T}_1, P)$  is a first countable  $\varepsilon$ -soft topological space,  $(\tilde{Y}, \mathcal{T}_2, P)$  is an  $\varepsilon$ -soft topological space and  $f : SE(\tilde{X}) \rightarrow SE(\tilde{Y})$  is a soft homeomorphism. Let's show that  $(\tilde{Y}, \mathcal{T}_2, P)$  is the first countable soft space. For any soft element  $\tilde{y} \in \tilde{Y}$ , since  $f$  is a bijection we have that for any  $\tilde{y} \in \tilde{Y}$  that  $f(\tilde{x}) = \tilde{y}$  for a soft element  $\tilde{x} \in \tilde{X}$ . Since  $(\tilde{X}, \mathcal{T}_1, P)$  is the first countable soft space, for each soft open set  $U \in \mathcal{T}_1$  with  $\tilde{x} \in B_n \subset U$ , there exists a countable soft local base  $B(\tilde{x}) = \{B_n : n \in \mathbb{N}\}$  of the  $\varepsilon$ -soft topology  $\mathcal{T}_1$ . Since  $f$  is soft open, there are soft open sets  $SS(f(SE(B_n))) = B'_n$  and  $SS(f(SE(U))) = V$  with  $f(\tilde{x}) \in f(SE(B_n)) \subset f(SE(U))$ . Hence,  $\tilde{y} = f(\tilde{x}) \in B'_n \subset V$ . Since  $f$  is surjective and  $B(\tilde{x})$  is a countable soft local base at  $\tilde{x} \in \tilde{X}$ ,  $B'(\tilde{y}) = \{B'_n : n \in \mathbb{N}\}$  is a countable soft local base at  $\tilde{y} \in \tilde{Y}$ . Therefore,  $(\tilde{Y}, \mathcal{T}_2, P)$  is the first countable  $\varepsilon$ -soft topological space.  $\square$

**Example 5.** The finite complement  $\varepsilon$ -soft topological space is not first countable.

### 3.2 | Second countable soft spaces

**Definition 11.** We call an  $\varepsilon$ -soft topological space  $(\tilde{X}, \mathcal{T}, P)$  a second countable soft space if there exists a countable soft base for the  $\mathcal{T}$ .

**Example 6.** Assume that  $P$  is a finite parameters set and  $(\tilde{\mathbb{R}}, \mathcal{T}_d, P)$  is an  $\varepsilon$ -soft real metric topological space. A class

$$B = \{B = (\tilde{x}, \tilde{y}) : \tilde{x} \tilde{z} \tilde{y}, \tilde{x}(\alpha), \tilde{y}(\alpha) \in \mathbb{Q}, \forall \alpha \in P\}$$

is a soft base for the  $\mathcal{T}_d$ . Because a class

$$B(\alpha) = \{B(\alpha) = (\tilde{x}, \tilde{y})(\alpha) = (\tilde{x}(\alpha), \tilde{y}(\alpha)), \forall \alpha \in P\}$$

is a base for the usual topological space  $(\mathbb{R}, \mathcal{T}_{d\alpha})$ .

**Example 7.** Assume that  $P$  is a finite parameters set. The discrete soft space  $(\tilde{\mathbb{R}}, \mathcal{P}(\tilde{\mathbb{R}}), P)$  is not a second countable space. A class  $B = \{ \{\tilde{x}\} : \tilde{x} \in \mathbb{R}(A) \}$  is a soft base for the discrete soft topology  $\mathcal{P}(\tilde{\mathbb{R}})$ . But since  $\mathbb{R}(A)$  is non-countable,  $B$  is non-countable.

*Remark 2.* A first countable soft space does not need to be a second countable soft space. For example, the discrete soft topology  $(\tilde{\mathbb{R}}, \mathcal{P}(\tilde{\mathbb{R}}), P)$  is a first countable soft space but is not a second countable soft space.

**Theorem 4.** Every second countable  $\varepsilon$ -soft topological space is also first countable.

*Proof.* Assume that  $(\tilde{X}, \mathcal{T}, P)$  is a second countable  $\varepsilon$ -soft topological space and  $\mathcal{B}$  is a soft base of the  $\varepsilon$ -soft topology  $\mathcal{T}$ . Then, the class  $\mathcal{B}(\tilde{x}) = \{B \in \mathcal{B} : \tilde{x} \in B\}$  is a soft local base for each  $\tilde{x} \in \tilde{X}$ .  $\mathcal{B}$  is countable also  $\mathcal{B}(\tilde{x})$  is countable. Thus,  $(\tilde{X}, \mathcal{T}, P)$  is a first countable soft space.  $\square$

**Theorem 5.** Every soft subspace of second countable  $\varepsilon$ -soft topological space is also second countable.

*Proof.* Assume that  $(\tilde{X}, \mathcal{T}, P)$  is a second countable  $\varepsilon$ -soft topological space,  $F \in S(\tilde{X})$  is a soft subset and  $(F, \mathcal{T}_F, P)$  is an  $\varepsilon$ -soft subspace. Since  $(\tilde{X}, \mathcal{T}, P)$  is the second countable, there is a countable soft base  $\mathcal{B}$ . Hence, a class  $\mathcal{B}_F = \{F \cap B : B \in \mathcal{B}, F \cap B \in S(\tilde{X})\}$  is a countable soft base for the  $\varepsilon$ -soft topology  $\mathcal{T}_F$ . Thus,  $(F, \mathcal{T}_F, P)$  is a second countable.  $\square$

**Theorem 6.** The property of being a second countable is a topological property.

*Proof.* Assume that  $(\tilde{X}, \mathcal{T}_1, P)$  is a second countable  $\varepsilon$ -soft topological space,  $(\tilde{Y}, \mathcal{T}_2, P)$  is an  $\varepsilon$ -soft topological space and  $f : SE(\tilde{X}) \rightarrow SE(\tilde{Y})$  is a soft homeomorphism. We need to show that  $(\tilde{Y}, \mathcal{T}_2, P)$  is a second countable. Since  $(\tilde{X}, \mathcal{T}_1, P)$  is the second countable, for all soft open sets  $U \in \mathcal{T}_1$  and all soft elements  $\tilde{x} \in U$ , there is  $B_n \in \mathcal{B}$  with  $\tilde{x} \in B_n \subset U$ . Hence, there is a countable soft base  $\mathcal{B} = \{B_n : n \in \mathbb{N}\}$  of the  $\mathcal{T}_1$ . Since  $f$  is soft continuous, for each  $f(\tilde{x}) \in f(SE(U))$ , there is a soft open set  $B_0 \in \mathcal{B}$  with  $f(\tilde{x}) \in f(SE(B_0)) \subset f(SE(U))$ . Also,  $f$  is a soft open. So, soft sets  $SS(f(SE(B_0))) = B'_0$  and  $SS(f(SE(U))) = V$  are soft open sets. Since  $f$  is soft open and  $\mathcal{B}$  is the soft open class, for every  $B_n \in \mathcal{B}$ , the soft set  $SS(f(SE(B_n))) = B'_n$  is a soft open set and for each soft open sets  $V \in \mathcal{T}_2$ ,  $B'_n \subset V$ . Since  $f$  is bijective and  $\mathcal{B}$  is countable,  $\mathcal{B}' = \{B'_n : n \in \mathbb{N}\}$  is countable. From  $f$  is the surjective,  $\mathcal{B}'$  is a countable soft base for the  $\mathcal{T}_2$ . Thus,  $(\tilde{Y}, \mathcal{T}_2, P)$  is the second countable.  $\square$

**Definition 12.** Assume that  $(\tilde{X}, \mathcal{T}, P)$  is an  $\varepsilon$ -soft topological space and  $F$  is a soft set of  $S(\tilde{X})$ . We call the class  $\mathcal{G} = \{U_i \in S(\tilde{X}) : i \in I\} \subset S(\tilde{X})$  a soft cover of the soft set  $F$  iff  $F \subset \cup_{i \in I} U_i$  and call  $\mathcal{G}$  a soft open cover of the soft set  $F$  if the class  $\mathcal{G} = \{U_i \in S(\tilde{X}) : U_i \in \mathcal{T}, i \in I\} \subset \mathcal{T}$ .

**Definition 13.** Assume that  $\mathcal{G}$  is a soft open cover of the soft set  $F$ . We call a class  $\mathcal{G}_n = \{U_n : n \in \mathbb{N}\}$  a countable soft subcover of  $\mathcal{G}$  iff  $F \subset \cup_{n \in \mathbb{N}} U_n$ .

**Theorem 7.** Assume that  $F \in S(\tilde{X})$  is a soft set of a second countable  $\varepsilon$ -soft topological space  $(\tilde{X}, \mathcal{T}, P)$ . If  $\mathcal{G}$  is a soft open cover of the  $F$ , then there is a countable soft subcover of the soft open cover  $\mathcal{G}$ .

*Proof.* Assume that  $\mathcal{B}$  is a countable soft base for the  $\mathcal{T}$ . Since  $F \subset \cup \{U : U \in \mathcal{G}\}$ , there is a soft set  $U_{\tilde{x}} \in \mathcal{G}$  with  $\tilde{x} \in U_{\tilde{x}}$  for every  $\tilde{x} \in F$ . Since  $\mathcal{B}$  is the soft base, there is a  $B_{\tilde{x}} \in \mathcal{B}$  such that  $\tilde{x} \in B_{\tilde{x}} \subset U_{\tilde{x}}$ . Hence,  $F \subset \cup \{B_{\tilde{x}} : \tilde{x} \in F\}$ . But  $\{B_{\tilde{x}} : \tilde{x} \in F\} \subset \mathcal{B}$ , so it is countable. Thus, we can say that  $\{B_{\tilde{x}} : \tilde{x} \in F\} \subseteq \{B_n : n \in \mathbb{N}\}$ . For each  $n \in \mathbb{N}$ , let's choose a soft set  $U_n \in \mathcal{G}$  such that  $B_n \subset U_n$ . Then  $F \subset \cup \{B_n : n \in \mathbb{N}\} \subset \cup \{U_n : n \in \mathbb{N}\}$ . Therefore,  $\mathcal{G}_n = \{U_n : n \in \mathbb{N}\}$  is a countable soft subcover of  $\mathcal{G}$ .  $\square$

## 4 | SOFT SEPARABILITY

### 4.1 | Separable soft spaces

**Definition 14.** Assume that  $(\tilde{X}, \mathcal{T}, P)$  is an  $\varepsilon$ -soft topological space and  $F, G \in S(\tilde{X})$  are two non-empty soft sets.

We call  $F$  dense soft set in itself if it is a soft subset of the soft limit set  $F'$ .

We call  $F$  dense soft set in  $G$  if the soft closure  $\bar{F}$  is a soft subset of  $G$ .

We call  $F$  nowhere dense soft set in  $(\tilde{X}, \mathcal{T}, P)$  if  $\bar{F}^\circ = \Phi$ .

We call  $F$  everywhere dense soft set in  $(\tilde{X}, \mathcal{T}, P)$  if  $\bar{F} = \tilde{X}$ .

**Example 8.** Let  $P = \{\alpha, \beta\}$  be a parameters set and  $X = \{x, y, z, t\}$ .  $SE(\tilde{X}) = \{\tilde{x}_i : i = 1, 2, \dots, 16\}$  where,

$$\begin{aligned} \tilde{x}_1 &= \{(\alpha, \{x\}), (\beta, \{x\})\}, & \tilde{x}_5 &= \{(\alpha, \{y\}), (\beta, \{x\})\}, & \tilde{x}_9 &= \{(\alpha, \{z\}), (\beta, \{x\})\}, & \tilde{x}_{13} &= \{(\alpha, \{t\}), (\beta, \{x\})\}, \\ \tilde{x}_2 &= \{(\alpha, \{x\}), (\beta, \{y\})\}, & \tilde{x}_6 &= \{(\alpha, \{y\}), (\beta, \{y\})\}, & \tilde{x}_{10} &= \{(\alpha, \{z\}), (\beta, \{y\})\}, & \tilde{x}_{14} &= \{(\alpha, \{t\}), (\beta, \{y\})\}, \\ \tilde{x}_3 &= \{(\alpha, \{x\}), (\beta, \{z\})\}, & \tilde{x}_7 &= \{(\alpha, \{y\}), (\beta, \{z\})\}, & \tilde{x}_{11} &= \{(\alpha, \{z\}), (\beta, \{z\})\}, & \tilde{x}_{15} &= \{(\alpha, \{t\}), (\beta, \{z\})\}, \\ \tilde{x}_4 &= \{(\alpha, \{x\}), (\beta, \{t\})\}, & \tilde{x}_8 &= \{(\alpha, \{y\}), (\beta, \{t\})\}, & \tilde{x}_{12} &= \{(\alpha, \{z\}), (\beta, \{t\})\}, & \tilde{x}_{16} &= \{(\alpha, \{t\}), (\beta, \{t\})\}. \end{aligned}$$

$\mathcal{T} = \{\Phi, F_1, F_2, F_3, F_4, F_5, F_6, F_7, \tilde{X}\}$  is an  $\varepsilon$ -soft topology over  $\tilde{X}$ , where,

$$\begin{aligned} F_1 &= \{(\alpha, \tilde{X}), (\beta, \{x, y, z\})\}, & F_5 &= \{(\alpha, \{x, z, t\}), (\beta, \{y, z, t\})\}, \\ F_2 &= \{(\alpha, \{y, z, t\}), (\beta, \tilde{X})\}, & F_6 &= \{(\alpha, \{z, t\}), (\beta, \{y, z, t\})\}, \\ F_3 &= \{(\alpha, \{y, z, t\}), (\beta, \{x, y, z\})\}, & F_7 &= \{(\alpha, \{z, t\}), (\beta, \{y, z, t\})\}, \\ F_4 &= \{(\alpha, \{x, z, t\}), (\beta, \{y, z, t\})\}, & & \end{aligned}$$

According to  $\mathcal{T}$  topology, the class of closed soft sets is  $\mathcal{K} = \{\Phi, K_3, K_4, K_5, K_6, K_7, \tilde{X}\}$  where,

$$\begin{aligned} K_3 &= \{(\alpha, \{x\}), (\beta, \{t\})\}, & K_6 &= \{(\alpha, \{x, y\}), (\beta, \{x\})\}, \\ K_4 &= \{(\alpha, \{y\}), (\beta, \{x\})\}, & K_7 &= \{(\alpha, \{x, y\}), (\beta, \{x, t\})\}, \\ K_5 &= \{(\alpha, \{y\}), (\beta, \{x, t\})\}, & & \end{aligned}$$

Let  $F = \{(\alpha, \{x\}), (\beta, \{x, t\})\}$  and  $G = \{(\alpha, \{x, y, z\}), (\beta, \{x, t\})\}$  be two soft sets of  $S(\tilde{X})$ .  $\bar{F} = \{(\alpha, \{x, y\}), (\beta, \{x, t\})\} = K_7$  and  $\bar{F} \tilde{C} G$ . Namely,  $F$  is a dense soft set in  $G$ .  $F$  is a nowhere dense soft set in  $(\tilde{X}, \mathcal{T}, P)$  since  $\bar{F}^\circ = \Phi$ .  $G$  soft set is an everywhere dense soft set in  $(\tilde{X}, \mathcal{T}, P)$  since  $\bar{G} = \tilde{X}$ .

**Remark 3.** Although soft interior of a soft set is empty, it can be a dense soft set in another soft set or everywhere dense soft set in  $\tilde{X}$ . For example, the soft interior  $G^\circ$  of the soft set  $G$  in Example 8 is soft empty, but  $G$  is an everywhere dense soft set in  $(\tilde{X}, \mathcal{T}, P)$ .

**Definition 15.** Assume that  $(\tilde{X}, \mathcal{T}, P)$  is an  $\varepsilon$ -soft topological space. We call a soft set  $F \in S(\tilde{X})$  a countable soft set if  $SE(F)$  is countable.

Assume that  $\mathcal{B} \subset SE(\tilde{X})$  is a class of soft elements. We call  $F = SS(\mathcal{B}) \in S(\tilde{X})$  a countable soft set if  $\mathcal{B}$  is countable.

**Definition 16.** We call any  $\varepsilon$ -soft topological space  $(\tilde{X}, \mathcal{T}, P)$  separable if there is a countable everywhere dense soft set  $F \in S(\tilde{X})$ .

**Example 9.** Assume that  $(\tilde{X}, \mathcal{T}, P)$  is the separable soft space in Example 8. In fact, the soft set  $G$  is countable and everywhere dense soft set in the soft set  $\tilde{X}$ .

**Example 10.** Assume that  $P$  is a finite parameters set and  $(\tilde{\mathbb{R}}, \mathcal{T}_d, P)$  is the  $\varepsilon$ -soft real metric topological space which produced by  $(\tilde{\mathbb{R}}, d, P)$ , for every  $\alpha \in P$  and every  $\tilde{x}, \tilde{y} \in \tilde{\mathbb{R}}$  defined  $d(\tilde{x}, \tilde{y})(\alpha) = |\tilde{x}(\alpha) - \tilde{y}(\alpha)|$  and that meets the axiom M5. Let  $\tilde{\mathbb{Q}} \in S(\tilde{\mathbb{R}})$  be the soft rational numbers set which defined  $\tilde{x}(\alpha) \in \mathbb{Q}$  for each  $\tilde{x} \in \tilde{\mathbb{Q}}$  and each  $\alpha \in P$ . Since  $\tilde{\mathbb{Q}} = \tilde{\mathbb{R}}$ , the  $\varepsilon$ -soft topological space  $(\tilde{\mathbb{R}}, \mathcal{T}, P)$  is separable.

**Example 11.** The discrete  $\varepsilon$ -soft topological space  $(\tilde{\mathbb{R}}, \mathcal{P}(\tilde{\mathbb{R}}), P)$  is not separable. Because every soft subset of  $S(\tilde{\mathbb{R}})$  is both soft closed and soft open. Hence, a soft set which the closure set equal to  $\tilde{\mathbb{R}}$  is  $\tilde{\mathbb{R}}$  itself. So  $SE(\tilde{\mathbb{R}}) = \mathbb{R}(A)$  is not separable.

**Theorem 8.** Every second countable  $\varepsilon$ -soft topological space is also separable.

*Proof.* Assume that  $\mathcal{B} = \{B_n : n \in \mathbb{N}\}$  is a countable soft base of a second countable  $\varepsilon$ -soft topological space  $(\tilde{X}, \mathcal{T}, P)$ . Let's choose a  $\tilde{x}_n \in B_n$  for each  $n \in \mathbb{N}$ . Then the soft set  $F = SS(\{\tilde{x}_n : n \in \mathbb{N}\}) \subset S(\tilde{X})$  is countable. Now, we show that  $\bar{F} = \tilde{X}$ . Since  $F \tilde{C} \bar{F}$  and  $F \cup F^C = \tilde{X}$ , if  $F \cup F^C = \bar{F}$  or equivalently if  $\tilde{x} \in \bar{F}$  for each soft element  $\tilde{x} \in F^C$ ,  $(\tilde{X}, \mathcal{T}, P)$  is separable. Since  $\bar{F} = F \cup F'$ , we need to show that  $\tilde{x} \in F'$  for each soft element  $\tilde{x} \in F^C$ . Let  $U$  be soft open set containing the soft element  $\tilde{x}$ . Then  $U$  includes at least one soft set  $B_{n_0} \in \mathcal{B}$ . Hence, there is  $\tilde{x}_{n_0} \in \tilde{X}$  with  $\tilde{x}_{n_0} \in B_{n_0} \tilde{C} U$ .  $\tilde{x}_{n_0} \neq \tilde{x}$  and so  $\tilde{x}_{n_0} \in F$ . Thus  $\tilde{x} \in F'$ .  $\square$

**Remark 4.** The converse of Theorem 8 is not true. Namely a separable  $\varepsilon$ -soft topological space cannot be second countable. For example, is an  $\varepsilon$ -soft topological soft space with the topology  $\mathcal{T} = \{G = (\tilde{x}, \tilde{y}) : \forall \alpha \in P, \tilde{x}(\alpha), \tilde{y}(\alpha) \in \mathbb{R}\}$

$\tilde{\mathbb{R}}, \tilde{x}(\alpha) < \tilde{y}(\alpha)\}$ . For each  $\alpha \in A$  the topological space  $(\mathbb{R}, \mathcal{T}_\alpha)$  is separable but is not second countable. Thus, the soft space  $(\tilde{\mathbb{R}}, \mathcal{T}, A)$  is separable but not second countable.

**Theorem 9.** Every separable  $\varepsilon$ -soft metric space that meets the axiom M5 is also a second countable soft space.

*Proof.* Assume that  $(\tilde{X}, d, P)$  is a soft metric space that meets the axiom M5. Then  $(\tilde{X}, \mathcal{T}_d, P)$  is an  $\varepsilon$ -soft topological space. Since  $\tilde{X}$  is separable then there exists a countable dense soft set  $F \in S(\tilde{X})$ . Let  $\mathcal{B} = \{SS(B(\tilde{x}, \tilde{r})) : \tilde{x} \in F, \tilde{r} > \tilde{0}, \tilde{r} \in \tilde{\mathbb{Q}}\}$  be the class of all soft open balls with center  $\tilde{x}$  and radius of  $\tilde{r}$ . Then the class  $\mathcal{B}$  is countable. We claim that the class  $\mathcal{B}$  is a soft base for the topology  $\mathcal{T}_d$ . Therefore, let's show that there exists at least one soft open ball  $SS(B(\tilde{x}, \tilde{r})) \in \mathcal{B}$  such that  $\tilde{y} \in B(\tilde{x}, \tilde{r}) \subset SE(U)$  for every  $U \in \mathcal{T}_d$  and every  $\tilde{y} \in U$ . Since  $\tilde{y} \in SE(U)$  there is a soft open ball  $B(\tilde{y}, \tilde{r})$  with  $\tilde{y} \in B(\tilde{y}, \tilde{r}) \subset SE(U)$ . Since  $F$  is dense in  $\tilde{X}$ ,  $F$  have soft element more than one. Hence there is a soft element  $\tilde{y} \in F$  such that  $d(\tilde{x}, \tilde{y}) < \frac{\tilde{r}}{3}$ . From the axiom M4 of the  $\varepsilon$ -soft metric space  $(\tilde{X}, d, P)$ , we can take soft number  $\tilde{r}_0 \in \tilde{\mathbb{Q}}^*$  such that  $\frac{\tilde{r}}{3} < \tilde{r}_0 < \frac{2\tilde{r}}{3}$ . Then there is an open ball  $B(\tilde{x}, \tilde{r}_0)$  such that  $B(\tilde{x}, \tilde{r}_0) \subset B(\tilde{x}, \tilde{r}) \subset SE(U)$ . Since  $SS(B(\tilde{x}, \tilde{r})) \in \mathcal{B}$ ,  $SS(B(\tilde{x}, \tilde{r}_0)) \in \mathcal{B}$ . Thus  $\mathcal{B}$  is a countable soft base for the soft topology  $\mathcal{T}_d$ .  $\square$

**Theorem 10.** The property of being a separable is a topological property.

*Proof.* Assume that  $(\tilde{X}, \mathcal{T}_1, P)$  is a separable  $\varepsilon$ -soft topological space,  $(\tilde{Y}, \mathcal{T}_2, P)$  is an  $\varepsilon$ -soft topological space and  $f : SE(\tilde{X}) \rightarrow SE(\tilde{Y})$  is a soft homeomorphism. Since  $(\tilde{X}, \mathcal{T}_1, P)$  is separable, there is a countable everywhere dense soft set  $F \in S(\tilde{X})$ . Then  $SE(F)$  is countable. Since  $f$  is bijective,  $f(SE(F)) \subset SE(\tilde{Y})$  is countable, i.e. the soft set  $G = SS(f(SE(F))) \in S(\tilde{Y})$  is countable. Since  $f$  is the soft homeomorphism  $f(clF) = cl(SS(f(SE(F)))) = clG$ . Since  $\tilde{F} = \tilde{X}$  and  $f$  is surjective,  $\tilde{G} = \tilde{Y}$ . Thus  $(\tilde{Y}, \mathcal{T}_2, P)$  is separable.  $\square$

*Remark 5.* Separable soft space is not hereditary property. The  $\varepsilon$ -soft space  $(\tilde{\mathbb{R}}, \mathcal{T}, P)$  in Example 10 is separable. But the soft subspace  $(\tilde{\mathbb{Q}}, \mathcal{T}_{\tilde{\mathbb{Q}}}, P)$  is not separable. Because for every infinite soft sets  $F \in S(\tilde{\mathbb{Q}})$ , we can write  $F = \tilde{\mathbb{Q}} \cap SS([\tilde{a}, \tilde{b}])$ ,  $F = \tilde{\mathbb{Q}} \cap SS([\tilde{a}, \tilde{b}])$  or  $F = \tilde{\mathbb{Q}} \cap SS([\tilde{a}, \tilde{b}])$ . According to the topology  $\mathcal{T}_{\tilde{\mathbb{Q}}(\alpha)}$ , for every  $\alpha \in P$  the soft set  $F(\alpha) = \tilde{\mathbb{Q}} \cap SS([\tilde{a}, \tilde{b}])(\alpha) = \tilde{\mathbb{Q}}(\alpha) \cap SS([\tilde{a}(\alpha), \tilde{b}(\alpha)]) = \mathbb{Q} \cap [a, b]$  is a subset of  $\mathbb{Q}$ . Since  $\tilde{F}(\alpha) = \overline{\mathbb{Q} \cap [a, b]} = [a, b] \neq \mathbb{Q}$ , there is not a countable dense soft subset of  $\mathbb{Q}$ . So  $\mathbb{Q}$  is not separable, i.e.  $(\tilde{\mathbb{Q}}, \mathcal{T}_{\tilde{\mathbb{Q}}}, P)$  is not separable.

**Example 12.** The finite complement  $\varepsilon$ -soft topological space  $(\tilde{X}, \mathcal{T}, P)$  is separable.

*Proof.* The finite complement  $\varepsilon$ -soft topology over  $\tilde{X}$  is

$$\mathcal{T} = \{\Phi, U \in S(\tilde{X}) : U^C \text{ is finite and } U^C \in S(\tilde{X})\}.$$

If  $SE(\tilde{X})$  countable, then clearly  $\tilde{X}$  is countable dense soft subset of  $(\tilde{X}, \mathcal{T}, P)$ . So  $(\tilde{X}, \mathcal{T}, P)$  is separable. Conversely, we suppose that  $SE(\tilde{X})$  is not countable. Then,  $S(\tilde{X})$  contains an infinite countable soft set  $F$ . According to the finite complement topology, the only  $\tilde{X}$  and finite soft sets of  $S(\tilde{X})$  are the soft closed sets. Hence, the soft closure of the infinite soft set  $F$  is the entire soft space  $\tilde{X}$ . Thus,  $(\tilde{X}, \mathcal{T}, P)$  is separable.  $\square$

**Example 13.** The discrete  $\varepsilon$ -soft topological space  $(\tilde{X}, \mathcal{T}, P)$  is separable if and only if  $SE(\tilde{X})$  is countable. In fact, every soft set  $F \in S(\tilde{X})$  is both soft closed and soft open. So, the only dense soft subset of  $\tilde{X}$  is  $\tilde{X}$  itself. Then  $\tilde{X}$  contains a countable dense soft subset iff  $SE(\tilde{X})$  countable.

## 4.2 | Lindelöf soft spaces

**Definition 17.** We call an  $\varepsilon$ -soft topological space  $(\tilde{X}, \mathcal{T}, P)$  Lindelöf soft space iff there is a countable soft subcover of every soft open cover of the soft set  $\tilde{X}$ .

**Example 14.** Every second countable soft space is a Lindelöf space. It is enough to take  $F = \tilde{X}$  from Theorem 7.

**Example 15.** If the soft set  $\tilde{X}$  is countable, the  $\varepsilon$ -soft topological space  $(\tilde{X}, \mathcal{T}, P)$  is a Lindelöf soft space.

**Example 16.** The soft space  $(\tilde{X}, \mathcal{P}(\tilde{X}), P)$  is a Lindelöf soft space if and only if the soft set  $\tilde{X}$  is countable.

**Example 17.** The discrete soft space  $(\tilde{\mathbb{R}}, \mathcal{P}(\tilde{\mathbb{R}}), P)$  is not Lindelöf soft space. Because the soft set  $\tilde{\mathbb{R}}$  is not countable.

**Theorem 11.** Every soft closed subspace of a Lindelöf soft space is also Lindelöf soft space.

*Proof.* Assume that  $(\tilde{X}, \mathcal{T}, P)$  is a Lindelöf  $\varepsilon$ -soft topological space and  $\{U_i : i \in I\}$  is a soft open cover of a soft closed set  $F \in S(\tilde{X})$ . Since  $F^C \in S(\tilde{X})$ ,  $F^C \in \mathcal{T}$  and  $\tilde{X} = F \cup F^C$ ,  $\mathcal{G} = \{F^C, U_i : i \in I\}$  is a soft open cover of the soft set  $\tilde{X}$ . Since  $(\tilde{X}, \mathcal{T}, P)$  is Lindelöf, there exists a countable soft subcover  $\mathcal{G}_n = \{F^C, U_n : n \in \mathbb{N}\}$  of  $\mathcal{G}$ . Therefore,  $\{U_n : n \in \mathbb{N}\}$  is a countable soft subcover of  $\{U_i : i \in I\}$ . Thus,  $F$  is a Lindelöf.  $\square$

**Theorem 12.** Assume that  $(\tilde{X}, \mathcal{T}_1, P)$  is a Lindelöf  $\varepsilon$ -soft topological space and  $(\tilde{Y}, \mathcal{T}_2, P)$  is an  $\varepsilon$ -soft topological space. If a soft mapping  $f : SE(\tilde{X}) \rightarrow SE(\tilde{Y})$  is soft continuous and surjective,  $(\tilde{Y}, \mathcal{T}_2, P)$  is Lindelöf.

*Proof.* We prove that the soft space  $SS(f(SE(\tilde{X}))) = \tilde{Y}$  is Lindelöf. Let  $\{U_i : i \in I\}$  be a soft open cover of  $\tilde{Y}$ . Since  $f$  is soft continuous, the soft set  $SS(f^{-1}(SE(U_i))) = V_i$  is a soft open set in  $(\tilde{X}, \mathcal{T}_1, P)$  for every  $U_i$ . Hence, the class  $\{V_i : i \in I\}$  is a soft open cover of  $\tilde{X}$ . Since  $\tilde{X}$  is the Lindelöf. So,  $\tilde{X} = \cup_{n \in \mathbb{N}} V_n$ . Namely, the class  $\{V_n : n \in \mathbb{N}\}$  is a countable soft subcover of  $\{V_i : i \in I\}$ . Then,  $SS(f(SE(V_n))) = U_n$  is a soft open set in  $(\tilde{Y}, \mathcal{T}_2, P)$  for every  $n \in \mathbb{N}$ . Since  $f$  surjective and  $\tilde{X} = \cup_{n \in \mathbb{N}} V_n$ ,  $\tilde{Y} = \cup_{n \in \mathbb{N}} U_n$ . Thus,  $\tilde{Y}$  is Lindelöf.  $\square$

**Corollary 1.** The property of being a Lindelöf  $\varepsilon$ -soft space is a topological property.

## CONFLICT OF INTEREST

This work does not have any conflicts of interest.

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