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Prediction of Crack Initiation Angle in Brittle Structures Containing Inclined Cracks

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Abstract—In computational fracture mechanics, prediction of crack initiation angle is a crucial role on determining the shape of incrementally growing crack profiles and resulting fatigue lives. In some fatigue problems, such as containing inclined cracks, depending on the types of loading, boundary conditions and problem geometry, cracks can grow in a curvilinear form. In this study, fracture behavior of different specimen types made of marble and PMMA are analyzed analytically in terms of prediction of crack initiation angle. Analytical predictions are carried out using a recently developed crack deflection angle equation for three different problems selected from the literature; a square plate containing an inclined central crack subjected to biaxial loading, a short beam bend (SBB) specimen containing an inclined edge crack subjected to three-point bending and an edge cracked triangular (ECT) specimen subjected to three-point bending. Comparison of obtained results with the available data from the literature showed close agreement. Thus, it is concluded that the recently developed criterion is capable and can be used to predict crack initiation angle for brittle materials with inclined cracks.

Keywords: Crack initiation angle, mixed mode fracture, crack deflection, inclined cracks

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1. INTRODUCTION

In many structures and industrial applications, fracture or fatigue crack problems are encountered due to many factors such as manufacturing method, material type, environmental conditions or different load types affecting the structure. Fatigue cracks, one of the fundamental causes of structural damages, are of great importance in the design and performance of engineering materials. The existing cracks in a structure can sometimes remain inside the structure up to a safe size or sometimes cause critical damages with a sudden fracture, and in some cases serious loss of life. To prevent such adverse situations, it is necessary to calculate and determine the fracture parameters in a precise and accurate manner in order to establish how long the structure can continue its function safely with the existing cracks, in other words, to establish the safe-life of the structures containing cracks. Therefore, recent studies with respect to ensuring a safe operation of the cracked structures have become more significant. Additionally, predicting the resistance and behavior of the damaged structure under different loading conditions during the design phase and improvements regarding the design at the preliminary analysis stage by taking into consideration such situations provide the opportunity to minimize possible damages.

Many studies performed on fracture and crack propagation problems in the literature have focused on structures subjected to pure mode-I loads in which the initial crack is perpendicular to the loading direction and propagates straight along its initial plane after loading. However, many cracked structures encountered in practical applications are subjected to different multi-axial loads i.e., directions of the loads are not perpendicular to the crack plane and the initial crack can incline or deflect from its initial plane with an angle. The approaches related to mode-I analysis are insufficient for these kinds of problems. Although the first studies on deflection of an initial crack from its plane started in the early 1960s with the study of Erdogan and Sih [1], the problems related to two-dimensional in-plane mixed mode fracture have been given much attention, especially in recent years. Maximum tangential stress (MTS) [1], minimum strain energy density (SED) [2, 3], maximum energy release rate (MERR) [4, 5], maximum tangential strain (MTSN) criteria [6] are some of the most popular and known criteria used to understand the fracture behavior of materials for in-plane mixed mode loading situations. Furthermore, Tanaka [7], Pook [8], and Richard [9] also developed different fracture criteria. It is seen from the studies in the literature that a wide variety of experimental and numerical analyses have been carried out using different types of specimens made of different materials for these types of in-plane mixed mode problems. Disc-type specimens [10–12], three or four-point bending specimens [13, 14], rectangular specimens with an angled corner or central crack [15, 16] are some of the most commonly used specimens. Moreover, different types of specimens were also examined in the literature such as compact tension shear (CTS) specimen [17–23], butterfly-type specimen [24–28], and T-specimen [19, 29] which are designed with its loading device to generate any combinations of modes I and II using conventional axial test machine.

Soda lime glass [16, 30], ceramic [31–33], polycrystalline graphite [34, 35], polymethyl methacrylate (PMMA) [36–40], concrete [41, 42], marble [43–45], aluminum alloys [46–48] and steels [49–51] are some kind of materials used as a specimen. Biner [52] carried out experimental tests on fatigue crack growth behavior of CTS specimen made of AISI-304 stainless steel and experimental crack deflection angles were compared with the predictions obtained using the SED criterion and the MERR criterion for different mode mixity cases. Although similar predictions were obtained for both criteria under low load mixities, the SED criterion considerably over-estimated the experimental crack deflection angle under high load mixities. Saghafi et al. [43] assessed the mixed mode fracture toughness of a marble rock using a semi-circular specimen with a vertical edge crack under a three-point bending test. Experimental data were compared with the MTS criterion and a modified maximum tangential stress (MMTS) criterion was proposed to predict the mixed mode fracture toughness since there was a high difference between the experimental results and the predictions made by the MTS criterion. Zafosnik et al. [53] performed numerical and experimental crack growth tests of AlMgMn4,5-W32 alloy using a CTS specimen under different in-plane mixed mode loading angle and crack propagation angles obtained from the experiments were compared with numerical predictions performed using the MTS and the SED criteria. It was seen from the comparisons that, the SED criterion was less accurate than the MTS criterion with respect to the determination of experimental crack kinking angle under high load mixities. Furthermore, with crack growth, the predictions by MTS criterion deviated from experimental results. Theoretical investigations on prediction of mixed mode fracture behavior of different materials such as soda lime glass, PMMA, rock materials, etc. were carried out in different studies by Mirsayar et al. [30, 38, 54] and by evaluating the existing traditional fracture criteria in the literature, an extended version of the MTSN criterion was proposed. It was reported from the studies that the proposed criterion by the authors provided more accurate predictions than the conventional fracture criteria. Similarly, mixed mode fracture toughness test results performed for the materials such as marble stone and PMMA were compared with the traditional fracture criteria by Hou et al. [39] and a new modified maximum energy release rate criterion was developed.

It should be noted that, although there have been numerous fracture criteria in the literature, there is no standard criterion for in-plane mixed mode loading and many of them are not sufficient for all mode mixities. Qian and Fatemi [55] and Ren et al. [56] surveyed various widely accepted mixed mode fracture criteria and they reported by referring to reviewed studies that discrepancies between the criteria and experimental findings were remarkable under high mode mixities in which mode-II component was dominant. In a previous study performed by the author et al. [29], in-plane mixed mode fracture experiments and numerical analyses were carried out, and similarly, it was observed from the results that, as is the case with the reports of the reviewed articles mentioned above, most traditional mixed mode fracture criteria deviated from experimental data under high mode mixity loading. Thus, an empirical mixed mode fracture criterion was developed in the study [29].

In computational fracture mechanics, the fracture criterion to be used to predict crack initiation angle has a key parameter on successive incrementally growing crack path and resulting fatigue life. This paper deals with the analytical investigations on fracture behavior of different specimen types made of marble and PMMA. In this context, crack initiation angle predictions are performed using a recently developed crack deflection angle equation for three different problems; a plate with an inclined central crack under biaxial loading, a short beam bend (SBB) specimen with an inclined edge crack under three-point bending and an edge cracked triangular (ECT) specimen under three-point bending.

2. ANALYTICAL MODEL

2.1. Traditional Crack Initiation Angle Equations

The crack initiation angle (θ_0) equation developed as part of maximum tangential stress (MTS) criterion by Erdogan and Sih [1] is one of the most widely used Eq. (2.1) for mixed mode fracture problems. Based on the criterion, crack extends from the crack tip radially and if the tangential stress reaches or exceeds a critical value, crack growth becomes unstable.

Table 1. Coefficients of the crack initiation angle equation (2.7)

a	*b*	*c*	*d*
0.1723	5.1062	–2.7483	–1.1636

$$\theta_0 = -\arccos\left(\frac{3K_{II}^2 + K_I\sqrt{K_I^2 + 8K_{II}^2}}{K_I^2 + 9K_{II}^2}\right), \tag{2.1}$$

where, θ_0 is the crack initiation angle, K_I and K_{II} are the mode-I and mode-II stress intensity factors, respectively.

The minimum strain energy density criterion (SED) which is based on the elastic energy density is an another common criterion developed by Sih [2, 3]. This criterion assumes that crack propagates in the direction of minimum strain energy density factor and the density factor reaches a critical value, crack propagation becomes unstable. Substituting the stress intensity factor, k_{I} and k_{II} into the following Eqs. (2.2) and (2.3), crack initiation angle, θ_0, can be obtained:

$$\begin{gathered}[2\cos\theta_0 - (\kappa-1)]\sin\theta_0 k_I^2 + 2[2\cos 2\theta_0 - (\kappa-1)\cos\theta_0]k_{\mathrm{I}}k_{\mathrm{II}} \\ +[(\kappa - 1 - 6\cos\theta_0)\sin\theta_0]k_{II}^2 = 0,\end{gathered} \tag{2.2}$$

$$\begin{gathered}[2\cos 2\theta_0 - (\kappa-1)\cos\theta_0]k_{\mathrm{I}}^2 + 2[(\kappa-1)\sin\theta_0 - 4\sin 2\theta_0]k_{\mathrm{I}}k_{\mathrm{II}} \\ +[(\kappa-1)\cos\theta_0) - 6\cos 2\theta_0]k_{\mathrm{II}}^2 > 0,\end{gathered} \tag{2.3}$$

where, $\kappa = 3 - 4v$ or $\kappa = (3 - v)/(1 + v)$ for plane strain or plane stress conditions, respectively.

The maximum energy release rate (MERR) criterion based on Griffith's theory [57] was expressed in several different forms by Nuismer [4] and Hussain [5]. Based on the criterion, crack extends in the direction of maximum strain energy release rate and becomes unstable if the rate exceeds a critical value. Crack initiation angles obtained from the MTS and the MERR criteria are identical.

Koo and Choy [58] developed the maximum tangential strain energy density (MTSE) criterion which is based on the tangential strain energy. According to the criterion, when the MTSE factor exceeds a critical value, crack initiation occurs in the direction of the density factor, C, which is defined by:

$$C = b_{11}k_1^2 + b_{12}k_1k_2 + b_{22}k_2^2 \tag{2.4}$$

in which coefficients are defined as follows;

$$\begin{gathered}b_{11} = (1/64\mu)(1 + \cos\theta)(\kappa + 2 + \cos\theta), \\ b_{12} = (1/64\mu)(\sin\theta - 3/2 - \kappa - 3\cos\theta), \\ b_{22} = (1/64\mu)(3\sin^2\theta)(\kappa + 3\cos\theta),\end{gathered} \tag{2.5}$$

where, $\kappa = 3 - 4\nu$ and $\kappa = (3 - \nu)/(1 + \nu)$ for plane strain or plane stress conditions, respectively, ν is Poisson's ratio and μ is the shear modulus. The crack initiation angle, θ_0 can be obtained by:

$$\frac{\partial C}{\partial\theta} = 0, \quad \frac{\partial^2 C}{\partial\theta^2} < 0 \quad \text{at which} \quad \theta = \theta_0. \tag{2.6}$$

2.2. The Recently Developed Crack Initiation Angle Equation

In a previous study [29], in-plane mixed mode fracture tests and computational analyses were carried out using different types of specimens such as CTS and T-specimen and obtained results from the experimental and numerical analyses were compared with the predictions by traditional fracture criteria and improved empirical fracture criteria were proposed. The recently developed empirical equation (2.7) to determine the crack initiation angle (θ_0) is given by;

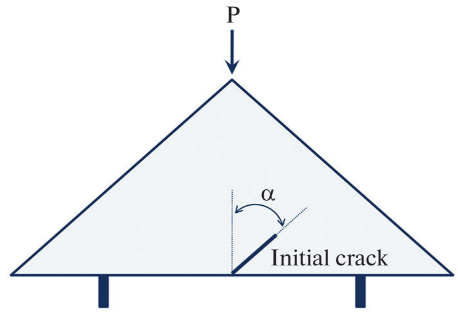

Fig. 1. Typical ECT specimen under three-point bending load.

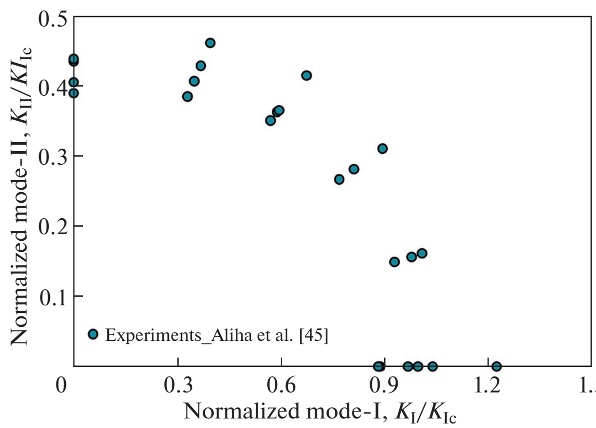

Fig. 2. Normalized mode-I and mode-II variations for different mode mixity ratios [45].

$$\theta_0 = -\arccos\left(\frac{aK_{II}^2 + K_I\sqrt{K_I^2 + bK_{II}K_I} + cK_{II}K_I}{K_I^2 + dK_{II}^2}\right). \tag{2.7}$$

Table 1 summarizes the coefficients of Eq. (2.7).

3. CRACK INITIATION PREDICTIONS

In this section, crack initiation predictions are carried out and obtained results are compared with the data from the literature. Aliha et al. [45] performed experimental and numerical mixed mode fracture tests on Neiriz marble rock material using edge cracked triangular (ECT) specimen containing an inclined edge crack. Figure 1 shows a typical ECT specimen under a three-point bending load. The initial crack angle (α) shown in the figure affects the mode mixity ratio of loading condition. By changing the specimen base length, the initial crack length, the distance between the two bottom supports and the initial crack angle, any combinations of modes-I and -II can be generated on the ECT specimen. Figure 2 represents the variation of normalized mode-I and mode-II values for different loading ratios obtained from experimental tests [45] in which obtained fracture toughness (K_{Ic}) value under pure mode-I loading ($\alpha = 0°$) was 1.23 MPa*m$^{1/2}$. The crack initiation angles corresponding to the crack deflection angles of initial cracks after loading are predicted using the mode-I and mode-II stress intensity factors. Figure 3 shows the variation of crack initiation angles as a function of mode mixity parameters, M^e, which is given by;

$$M^e = \frac{2}{\pi}\tan^{-1}\left(\frac{K_I}{K_{II}}\right). \tag{2.8}$$

It is seen from Fig. 3 that the predictions made by the Demir and Ayhan [29] and the MTS [1] criteria are almost identical for up to the highest level of mode mixity. Closer predictions are obtained between

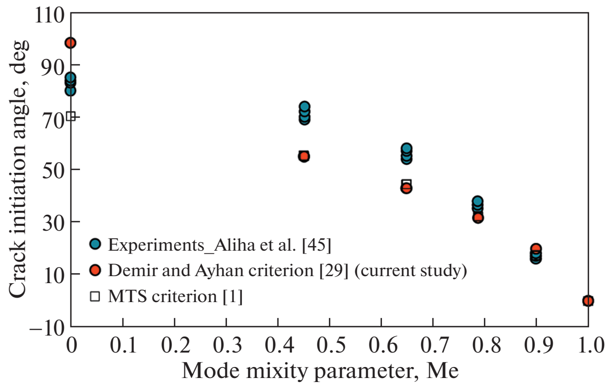

Fig. 3. Crack initiation angle vs. mode mixity parameter.

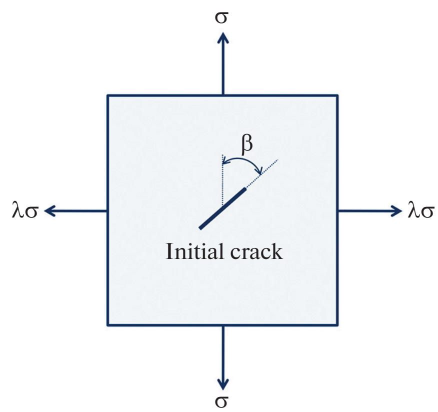

Fig. 4. A square plate containing an inclined central crack subjected to biaxial loading.

the criteria and the experimental data for lower mixed mode loading situations and the difference between the predictions and experimental measurements [45] increases with increasing the mode mixity ratio.

The study on a square plate made of PMMA containing an inclined central crack subjected to biaxial loading is considered as a second case study. In a previous study, theoretical investigations were performed and an extended version of the maximum tangential strain (EMTSN) criterion was proposed by Mirsayar [38] using the experimental data available in the literature [59, 60]. Figure 4 exhibits the loading configuration of the specimen. The central crack angle (β) shown in the figure is the primary component that determines the mode mixity ratio of loading condition together with the other loading and geometry parameters. The central crack is subjected to uniform far-field stresses and λ, shown in the figure symbolizes the lateral load ratio. Figure 5 illustrates the variation of normalized stress intensity factors for mode-I and mode-II obtained from the experiments [59, 60] under different mode mixity ratios. Analytical investigations are carried out using both of these experimental data given in the figure for PMMA. Comparison of crack initiation angle predictions according to the Demir and Ayhan [29] and the EMTSN criteria [38] with experimental findings are shown as a function of mode mixity parameter (M^e) in Fig. 6. As is seen from the figure that both of the criteria are in close agreement with the experimental measurements for up to pure mode-II loading condition ($M^e = 0$).

A short beam bend (SBB) specimen made of PMMA containing an inclined edge crack subjected to three-point bending load for which in-plane mixed mode experiments were performed by Mousavi et al. [40] is considered as a final case study. Figure 7 exhibits a general loading configuration of the SBB specimen. As is the previous case studies analyzed in this study, mixed mode load ratios can be changed by changing the angle of initial crack. Normalized mode-I and mode-II stress intensity factors obtained from the experiments [40] for different mode mixity ratios and using those of mixed mode stress intensity factors, comparison of crack initiation angle predictions with experimental data are given in Figs. 8 and 9, respectively. In Figure 9, a comparison graph is plotted as a function of precrack inclination angle which

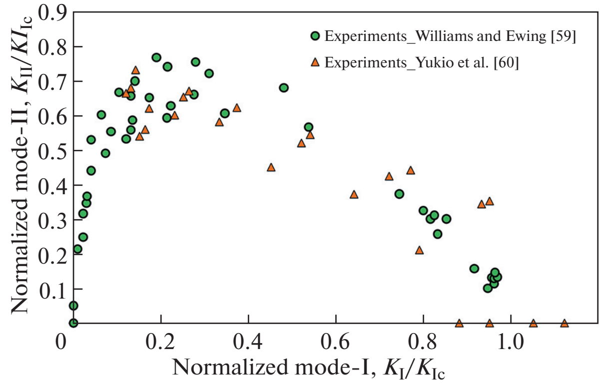

Fig. 5. Variations of normalized mode-I and mode-II stress intensity factors for different mode mixity ratios [38].

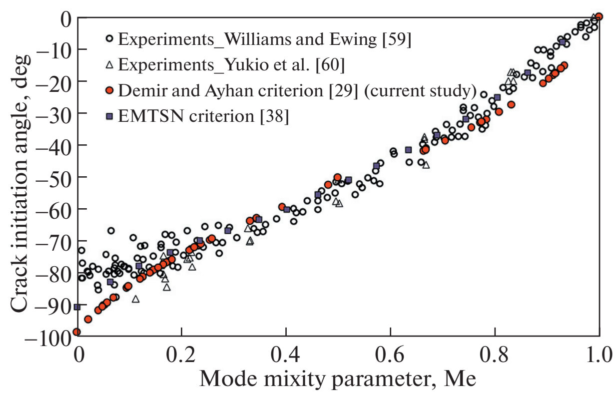

Fig. 6. Crack initiation angle predictions as a function of mode mixity parameter, M^e.

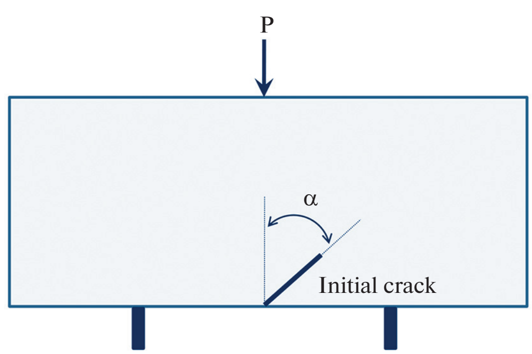

Fig. 7. Loading configuration of a short beam bend (SBB) specimen containing an inclined edge crack.

determines the ratio of mode mixity. According to the specimen configuration shown in Fig. 7, the loading situation is described as pure mode-I loading in case of $\alpha = 0°$ and pure mode-II loading is generated with the configuration for $\alpha = 39°$. It is seen from Fig. 9 that as is the case with the first case study closer predictions are obtained from the Demir and Ayan [29] and the MTS criteria [1] for up to the highest level of mode mixity. Similar results are also observed from the criteria considered in the second case study. The existing and the recently developed criteria predict the experimental measurements for pure mode-II loading with an average error rate of about 10–15% for all considered case studies. It should be noted that in practice, it is rare to encounter cracks subjected to pure mode-II loading.

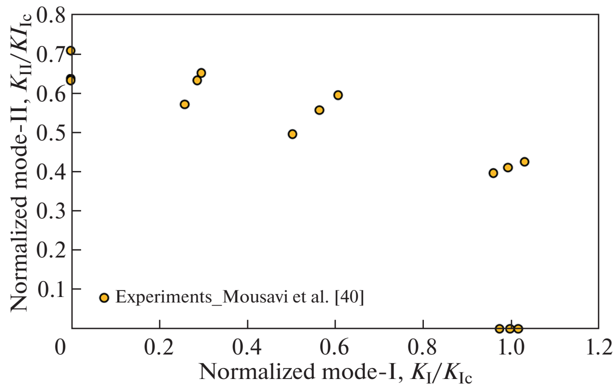

Fig. 8. Normalized mode-I and mode-II stress intensity factor variations for different mixed mode loading [40].

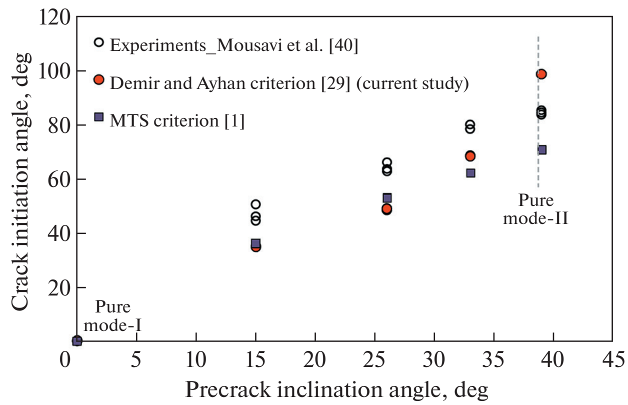

Fig. 9. Crack initiation angle predictions as a function of initial crack inclination angle.

4. CONCLUSIONS

The paper presents an analytical investigation for prediction of crack initiation angle of structures containing cracks. Fracture behavior of different specimen types made of marble and PMMA are analyzed in terms of crack initiation angle predictions. The applicability of the crack deflection angle equation which was recently developed using aluminum specimens by our research group is evaluated and discussed. Three different in-plane mixed mode problems selected from the literature; a square plate with an inclined central crack, a short beam bend (SBB) specimen containing an inclined edge crack and an edge cracked triangular (ECT) specimen with an inclined edge crack, all of which are subjected to different loading configuration and situations are studied.

Comparison of obtained results from these applications shows that the existing and the recently developed criteria are in excellent agreement with each other for up to highly mixed mode loading conditions. Remarkable differences exist between the predictions and the experimental findings for pure mode-II loading which is rare to encounter such these types of loading situations in engineering applications. It is concluded that the recently developed equation using aluminum specimens is also convenient for marble and PMMA and can be used to predict crack initiation angle for brittle materials with inclined cracks.

REFERENCES

1. F. Erdogan and G. C. Sih, "On the crack extension in plates under plane loading and transverse shear," J. Basic Eng. **85** (4), 519–525 (1963).
https://doi.org/10.1115/1.3656897
2. G. C. Sih and B. Macdonald, "Fracture mechanics applied to engineering problems-strain energy density fracture criterion," Eng. Fract. Mech. **6** (2), 361–386 (1974).
https://doi.org/10.1016/0013-7944(74)90033-2

3. G. C. Sih, "Strain-energy-density factor applied to mixed mode crack problems," Int. J. Fract. **10** (3), 305–321 (1974).
https://doi.org/10.1007/BF00035493
4. R. J. Nuismer, "An energy release rate criterion for mixed mode fracture," Int. J. Fract., **11** (2), 245–250 (1975).
https://doi.org/10.1007/BF00038891
5. M. A. Hussain, S. L. Pu, and J. Underwood, "Strain energy release rate for a crack under combined mode I and mode II," in *Fracture Analysis: Proceedings of the National Symposium on Fracture Mechanics, Maryland, USA, 1973*, Ed. by G. Irwin (ASTM International, West Conshohocken, 1974), Part II, pp. 2–28.
6. K. J. Chang, "On the maximum strain criterion – a new approach to the angled crack problem," Eng. Fract. Mech. **14** (1), 107–124 (1981).
https://doi.org/10.1016/0013-7944(81)90021-7
7. K. Tanaka, "Fatigue crack propagation from a crack inclined to the cyclic tensile axis," Eng. Fract. Mech. **6** (3), 499–507 (1974).
https://doi.org/10.1016/0013-7944(74)90007-1
8. L. P. Pook, "The significance of mode I branch cracks for mixed mode fatigue crack growth threshold behaviour," in *Biaxial and Multiaxial Fatigue,* Eds. by M. W. Brown and K. J. Miller (Mech. Engng. Publ., London, 1989), pp. 247–263.
9. H. A. Richard, "Safety estimation for construction units with cracks under complex loading," Int. J. Mater. Prod. Technol. **3** (3–4), 326–338 (1988).
10. N. Al-Shayea, "Crack propagation trajectories for rocks under mixed mode I-II fracture," Eng. Geol. **(81)** 1, 84–97 (2005).
https://doi.org/10.1016/j.enggeo.2005.07.013
11. M. R. M. Aliha, H. Behbahani, H. Fazaeli, and M. H. Rezaifar, "Study of characteristic specification on mixed mode fracture toughness of asphalt mixtures," Constr. Build. Mater. **54**, 623–635 (2014).
https://doi.org/10.1016/j.conbuildmat.2013.12.097
12. J. Akbardoost and A. Rastin, "Comprehensive data for calculating the higher order terms of crack tip stress field in disk-type specimens under mixed mode loading," Theor. Appl. Fract. Mech. **76**, 75–90 (2015).
https://doi.org/10.1016/j.tafmec.2015.01.004
13. M. R. M. Aliha, M. R. Ayatollahi, and B. Kharazi, "Numerical and experimental investigations of mixed mode fracture in granite using four-point-bend specimen," in *Damage and Fracture Mechanics*, Ed. by T. Boukharouba, M. Elboujdaini and G. Pluvinage (Springer, Dordrecht, 2009), pp. 275–283.
14. L. Marsavina, T. Sadowski, and M. Knec, "Crack propagation paths in four point bend Aluminium-PMMA specimens," Eng. Fract. Mech. **108**, 139–151 (2013).
https://doi.org/10.1016/j.engfracmech.2013.02.029
15. G. M. Seed and D. Nowell, "Use of the distributed dislocations method to determine the T-stress," Fatigue Fract. Eng. Mater. Struct. **17** (5), 605–618 (1994).
https://doi.org/10.1111/j.1460-2695.1994.tb00259.x
16. M. R. Ayatollahi and M. R. M. Aliha, "Analysis of a new specimen for mixed mode fracture tests on brittle materials," Eng. Fract. Mech. **76** (11), 1563–1573 (2009).
https://doi.org/10.1016/j.engfracmech.2009.02.016
17. H. A. Richard and K. Benitz, "A loading device for the creation of mixed mode in fracture mechanics," Int. J. Fract. **22** (2), R55-R58 (1983).
https://doi.org/10.1007/BF00942726
18. M. Sander and H. A. Richard, "Experimental and numerical investigations on the influence of the loading direction on the fatigue crack growth," Int. J. Fatigue **28** (5–6), 583–591 (2006).
https://doi.org/10.1016/j.ijfatigue.2005.05.012
19. O. Demir, A. O. Ayhan, S. Iric, and H. Lekesiz, "Evaluation of mixed mode-I / II criteria for fatigue crack propagation using experiments and modeling," Chinese J. Aeronaut. **31** (7), 1525–1534 (2018).
https://doi.org/10.1016/j.cja.2018.05.009
20. O. Demir, A. O. Ayhan, and H. Lekesiz, "Investigation of mixed mode - I/II fracture problems - Part 1: computational and experimental analyses," Frattura ed Integrità Strutturale **10** (35), 330–339 (2016).
https://doi.org/10.3221/IGF-ESIS.35.38
21. V. Patle, S. S. Bhadauria, and J. Abhishek, "Analysis of crack tip opening displacement under mixed mode fracture using fem technique," IOSR J. Mech. Civ. Eng. **3** (5), 27–34 (2012).
https://doi.org/10.9790/1684-0352734
22. J. Zhao and W. Guo, "Three-parameter K-T-Tz characterization of the crack-tip fields in compact-tension-shear specimens," Eng. Fract. Mech. **92**, 72–88 (2012).
https://doi.org/10.1016/j.engfracmech.2012.06.004
23. P. S. S. Gouda, K. G. Kodancha, and D. J. Siddaramaiah, "Experimental and numerical investigations on fracture behavior of high silica glass/satin textile fiber reinforced hybrid polymer composites," Adv. Mater. Lett. **4**

(11), 827–835 (2013).
https://doi.org/10.5185/amlett.2013.3450

24. B. Katanchi, N. Choupani, J. Khalil-Allafi, et al., “Mixed-mode fracture of a superelastic NiTi alloy: Experimental and numerical investigations,” Eng. Fract. Mech. **190**, 273–287 (2018).
https://doi.org/10.1016/j.engfracmech.2017.12.027
25. F. Tariq, M. Khan, M. Farhan, and M. Z. Siddiqui, “Strength of aluminum alloys under static mixed-mode i/ii loading conditions,” J. Test. Eval. **46** (1), 294–304 (2017).
https://doi.org/10.1520/JTE20160475
26. J. M. Greer Jr., S. E. G. Dorman, and M. J. Hammond, “Some comments on the Arcan mixed-mode (I/II) test specimen,” Eng. Fract. Mech. **78** (9), 2088–2094 (2011).
https://doi.org/10.1016/j.engfracmech.2011.03.017
27. I. Ud Din, P. Hao, S. Panier, et al., “Design of a new Arcan fixture for in-plane pure shear and combined normal/shear stress characterization of fiber reinforced polymer composites,” Exp. Tech. **44** (2), 231–240 (2020).
https://doi.org/10.1007/s40799-019-00353-9
28. G. P. Pucillo, M. Grasso, F. Penta, and P. Pinto, “On the mechanical characterization of materials by Arcan-type specimens,” Eng. Fract. Mech. **78** (8), 1729–1741 (2011).
https://doi.org/10.1016/j.engfracmech.2011.02.002
29. O. Demir, A. O. Ayhan, and S. İriç, “A new specimen for mixed mode-I/II fracture tests: Modeling, experiments and criteria development,” Eng. Fract. Mech. **178**, 457–476 (2017).
https://doi.org/10.1016/j.engfracmech.2017.02.019
30. M. M. Mirsayar, V. A. Joneidi, R. V. V Petrescu, et al., “Extended MTSN criterion for fracture analysis of soda lime glass,” Eng. Fract. Mech. **178**, 50–59 (2017).
https://doi.org/10.1016/j.engfracmech.2017.04.018
31. M. R. M. Aliha and M. R. Ayatollahi, “Analysis of fracture initiation angle in some cracked ceramics using the generalized maximum tangential stress criterion,” Int. J. Solids Struct. **49** (13), 1877–1883 (2012).
https://doi.org/10.1016/j.ijsolstr.2012.03.029
32. M. Zabihi, M. R. Ayatollahi, and H. R. Rezaie, “Mixed-mode fracture of synthesized nanocrystalline forsterite for biomedical applications,” Theor. Appl. Fract. Mech. **94**, 173–180 (2018).
https://doi.org/10.1016/j.tafmec.2018.02.003
33. S. Suresh, C. F. Shih, A. Morrone, and N. P. O’Dowd, “Mixed-mode fracture toughness of ceramic materials,” J. Am. Ceram. Soc. **73** (5), 1257–1267 (1990).
https://doi.org/10.1111/j.1151-2916.1990.tb05189.x
34. M. R. Ayatollahi and M. R. M. Aliha, “Mixed mode fracture analysis of polycrystalline graphite-a modified MTS criterion,” Carbon **46** (10), 1302–1308 (2008).
https://doi.org/10.1016/j.carbon.2008.05.008
35. M. M. Mirsayar, F. Berto, M. R. M. Aliha, and P. Park, “Strain-based criteria for mixed-mode fracture of polycrystalline graphite,” Eng. Fract. Mech. **156**, 114–123 (2016).
https://doi.org/10.1016/j.engfracmech.2016.02.011
36. M. R. Ayatollahi, M. R. M. Aliha, and M. M. Hassani, “Mixed mode brittle fracture in PMMA-An experimental study using SCB specimens,” Mater. Sci. Eng. A **417**, 348–356 (2006).
https://doi.org/10.1016/j.msea.2005.11.002
37. L. H. Hernandez-Gomez, I. Sauceda-Meza, G. Urriolagoitia-Calderón, et al., “Evaluation of crack initiation angle under mixed mode loading at diverse strain rates,” Theor. Appl. Fract. Mech. **42** (1), 53–61 (2004).
https://doi.org/10.1016/j.tafmec.2004.06.008
38. M. M. Mirsayar, “Mixed mode fracture analysis using extended maximum tangential strain criterion,” Mater. Des. **86**, 941–947 (2015).
https://doi.org/10.1016/j.matdes.2015.07.135
39. C. Hou, X. Jin, X. Fan, et al., “A generalized maximum energy release rate criterion for mixed mode fracture analysis of brittle and quasi-brittle materials,” Theor. Appl. Fract. Mech. **100**, 78–85 (2019).
https://doi.org/10.1016/j.tafmec.2018.12.015
40. S. S. Mousavi, M. R. M. Aliha, and D. M. Imani, “On the use of edge cracked short bend beam specimen for PMMA fracture toughness testing under mixed-mode I/II,” Polym. Test. **81**, 106199 (2020).
https://doi.org/10.1016/j.polymertesting.2019.106199
41. S. Seitl, P. Miarka, and V. Bilek, “The mixed-mode fracture resistance of C 50/60 and its suitability for use in precast elements as determined by the Brazilian disc test and three-point bending specimens,” Theor. Appl. Fract. Mech. **97**, 108–119 (2018).
https://doi.org/10.1016/j.tafmec.2018.08.003
42. Y. S. Jenq and S. P. Shah, “Mixed-mode fracture of concrete,” Int. J. Fract. **38** (2), 123–142 (1988).
https://doi.org/10.1007/BF00033002

43. H. Saghafi, M. R. Ayatollahi, and M. Sistaninia, "A modified MTS criterion (MMTS) for mixed-mode fracture toughness assessment of brittle materials," Mater. Sci. Eng. A **527** (21–22), 5624–5630 (2010). https://doi.org/10.1016/j.msea.2010.05.014

44. M. R. M. Aliha, M. R. Ayatollahi, and J. Akbardoost, "Typical upper bound – lower bound mixed mode fracture resistance envelopes for rock material," Rock Mech. Rock Eng. **45**, 65–74 (2012). https://doi.org/10.1007/s00603-011-0167-0

45. M. R. M. Aliha, G. R. Hosseinpour, and M. R. Ayatollahi, "Application of cracked triangular specimen subjected to three-point bending for investigating fracture behavior of rock materials," Rock Mech. Rock Eng. **46** (5), 1023–1034 (2013). https://doi.org/10.1007/s00603-012-0325-z

46. Y. A. Roy, R. Narasimhan, and P. R. Arora, "An experimental investigation of constraint effects on mixed mode fracture initiation in a ductile aluminium alloy," Acta Mater. **47** (5), 1587–1596 (1999). https://doi.org/10.1016/S1359-6454(99)00015-4

47. X. Qian and W. Yang, "Initiation of ductile fracture in mixed-mode I and II aluminum alloy specimens," Eng. Fract. Mech. **93**, 189–203 (2012). https://doi.org/10.1016/j.engfracmech.2012.06.018

48. Z. Husaini, "Mixed mode fracture behavior of an aluminum alloy A6061 investigated by using compact tension shear specimens," Int. J. Tech. **3**, 456–462 (2016). https://doi.org/10.14716/ijtech.v7i3.1924

49. C.-S. Kim and K.-W. Chung, "A study on fatigue crack propagation of rail steel under constant and mixed mode variable amplitude loadings," Int. J. Railw. **5** (2), 71–76 (2012). https://doi.org/10.7782/IJR.2012.5.2.071

50. N. Hallbäck, "Mixed-mode I/II fracture behaviour of a high strength steel," Int. J. Fract. **87** (4), 363–388 (1997). https://doi.org/10.1023/A:1007448511822

51. P. Kotowski, G. Lesiuk, J. A. F. O. Correia, and A. M. P. de Jesus, "Mixed mode (I+ II) fatigue crack paths in S355J0 steel in terms of fractal geometry," AIP Conf. Proc. **2028**, 020005 (2018). https://doi.org/10.1063/1.5066395

52. S. B. Biner, "Fatigue crack growth studies under mixed-mode loading," Int. J. Fatigue **23**, 259–263 (2001). https://doi.org/10.1016/S0142-1123(01)00146-3

53. B. Zafošnik, Z. Ren, M. Ulbin, and J. Flašker, "Evaluation of stress intensity factors using finite elements," in *FENET technology workshop, Trieste 2002*, 2002.

54. M. M. Mirsayar, A. Razmi, M. R. M. Aliha, and F. Berto, "EMTSN criterion for evaluating mixed mode I/II crack propagation in rock materials," Eng. Fract. Mech. **190**, 186–197 (2018).

55. J. Qian and A. Fatemi, "Mixed mode fatigue crack growth: A literature survey," Eng. Fract. Mech. **55** (6), 969–990 (1996). https://doi.org/10.1016/S0013-7944(96)00071-9

56. L. Ren, Z. Zhu, Q. Yang, and T. Ai, "Investigation on the applicability of several fracture criteria to the mixed mode brittle fractures," Adv. Mech. Eng. **5**, 545108 (2013). https://doi.org/10.1155/2013/545108

57. A. A. Griffith, "VI. The phenomena of rupture and flow in solids," Philos. Trans. R. Soc. London. Ser. A, **221** (582–593), 163–198 (1921). https://doi.org/10.1098/rsta.1921.0006

58. J. M. Koo and Y. S. Choy, "A new mixed mode fracture criterion: maximum tangential strain energy density criterion," Eng. Fract. Mech. **39** (3), 443–449 (1991). https://doi.org/10.1016/0013-7944(91)90057-8

59. J. G. Williams and P. D. Ewing, "Fracture under complex stress-the angled crack problem," Int. J. Fract. Mech. **8** (4), 441–446 (1972). https://doi.org/10.1007/BF00191106

60. U. Yukio, I. Kazuo, Y. Tetsuya, and A. Mitsuru, "Characteristics of brittle fracture under general combined modes including those under bi-axial tensile loads," Eng. Fract. Mech. **18** (6), 1131–1158 (1983). https://doi.org/10.1016/0013-7944(83)90007-3