DOI 10.1007/s11253-023-02161-2
Ukrainian Mathematical Journal, Vol. 74, No. 11, April, 2023 (Ukrainian Original Vol. 74, No. 11, November, 2022)

A NEW APPROACH TO THE APPROXIMATION BY POSITIVE LINEAR OPERATORS IN WEIGHTED SPACES

Ö. G. Atlihan,[1] **T. Yurdakadim,**[2,3] **and E. Taş**[4]

UDC 517.5

In the present paper, we deal with the problem of approximation of a function by positive linear operators in weighted spaces. Our main tool is the P_p-statistical convergence recently defined by [M. Ünver and C. Orhan, *Numer. Funct. Anal. Optim.*, **40**, 535–547 (2019)]. It is worth noting that the P_p-statistical convergence and statistical convergence do not imply each other.

1. Introduction

The classical Korovkin theorem establishes the uniform convergence of a sequence of positive linear operators to the identity operator in the space of real-valued continuous functions $C[0,1]$ by using only three functions. In other words, this type of theorems exhibits a variety of test functions, which assumes that the approximation property holds in the entire space if it holds for these functions. This property was discovered by Korovkin in 1953 for the functions $1,\ x$ and x^2 in $C[0,1]$ [15]. Due to the simplicity and efficiency of these theorems, the Korovkin-type approximation theory turned into a popular and well-studied part of the approximation theory.

By using various types of convergence or changing the test functions, numerous mathematicians investigated the Korovkin-type approximation theorems for a sequence of positive linear operators defined on different spaces [1, 3, 5, 8, 10, 14, 17]. In 2002, Gadjiev and Orhan gave a Korovkin-type approximation theorem based on the concept of statistical convergence. This concept of convergence is indeed efficient in applications because it turns a nonconvergent sequence into a convergent sequence. Later, this process was developed by Duman, et al. (see [7–10]) and appeared in the literature with the keyword statistical approximation. Recent results about the statistical approximation can be found in the monograph by Anastassiou and Duman [2]. Various convergence methods were also applied to the approximation in weighted spaces [4]. Recently, Ünver and Orhan [18] have introduced the notion of P_p-statistical convergence and, by using this convergence, proved a Korovkin-type theorem for a sequence of positive linear operators defined on $C[0,1]$. It is worth noting that the P_p-statistical convergences do not imply each other.

In the present paper, we prove a Korovkin-type approximation theorem for a sequence of positive linear operators acting from a weighted space C_{ϱ_1} into a weighted space B_{ϱ_2} with the use of the P_p-statistical convergence. Then we present an application, which shows that our new result is stronger than its classical version. We also establish the rate of convergence of these operators.

We now recall the concepts of weighted spaces considered in [4, 12]. As usual, a weight function $\varrho\colon \mathbb{R} \to \mathbb{R}$ is continuous on $\mathbb{R}$, nonincreasing on $(-\infty, 0)$, nondecreasing on $(0,\infty)$, $\varrho(0)=1$, and

$$\lim_{|x|\to\infty} \varrho(x) = \infty.$$

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[1] Pamukkale University, Denizli, Turkey; e-mail: oatlihan@pau.edu.tr.

[2] Bilecik Şeyh Edebali University, Bilecik, Turkey; e-mail: tugbayurdakadim@hotmail.com.

[3] Corresponding author.

[4] Kırşehir Ahi Evran University, Kırşehir, Turkey; e-mail: emretas86@hotmail.com.

Published in Ukrains'kyi Matematychnyi Zhurnal, Vol. 74, No. 11, pp. 1447–1453, November, 2022. Ukrainian DOI: 10.37863/umzh.v74i11.6427. Original article submitted May 12, 2020.

0041-5995/23/7411–1649

Then the corresponding weighted spaces are defined as follows:

$$B_\varrho := \{f\colon \mathbb{R} \to \mathbb{R}\colon |f| \le \varrho M_f \text{ on } \mathbb{R} \text{ for some } M_f > 0\},$$

$$C_\varrho := \{f\colon \mathbb{R} \to \mathbb{R}\colon f \in B_\varrho \text{ and } f \text{ is continuous on } \mathbb{R}\}.$$

It is well known that the spaces B_ϱ and C_ϱ are Banach spaces with the norm

$$|f|_\varrho = \sup_{x\in\mathbb{R}} \frac{|f(x)|}{\varrho(x)}.$$

Now let ϱ_1 and ϱ_2 be two weight functions. We also assume that the condition

$$\lim_{|x|\to\infty} \frac{\varrho_1(x)}{\varrho_2(x)} = 0 \tag{1}$$

holds. Then one can easily note that

$$C_{\varrho_1} \subset C_{\varrho_2} \qquad \text{and} \qquad B_{\varrho_1} \subset B_{\varrho_2}.$$

If T is a positive linear operator from C_{ϱ_1} into B_{ϱ_2}, then the operator norm is given by

$$\|T\|_{C_{\varrho_1}\to B_{\varrho_2}} := \sup_{|f|_{\varrho_1}=1} \|Tf\|_{\varrho_2} = \big\|T(\varrho_1)\big\|_{\varrho_2}.$$

The following approximation theorem for a sequence of positive linear operators acting from the weighted space C_{ϱ_1} into B_{ϱ_2} can be found in [12, 13]:

Theorem 1. *Let ϱ_1 and ϱ_2 be two weight functions satisfying (1). Assume that $\{T_n\}$ is a sequence of positive linear operators acting from C_{ϱ_1} into B_{ϱ_2}. If*

$$\lim_n \|T_n(F_v) - F_v\|_{\varrho_1} = 0,$$

where

$$F_v(x) = \frac{x^v \varrho_1(x)}{1+x^2}, \qquad v = 0, 1, 2,$$

then, for all $f \in C_{\varrho_1}$,

$$\lim_n \big\|T_n(f) - f\big\|_{\varrho_2} = 0.$$

Theorem 1 was studied in [12, 13] and its statistical version was obtained in [4, 8, 9]. We now recall the concept of P_p-statistical convergence, which is the primary concept in the present paper.

Let (p_j) be a real sequence with $p_0 > 0$ and $p_1,\ p_2, \ldots \ge 0$ such that the corresponding power series

$$p(t) := \sum_{j=0}^{\infty} p_j t^j$$

has the radius of convergence R and $0 < R \leq \infty$. If the limit

$$\lim_{t\to R^-} \frac{1}{p(t)} \sum_{j=0}^{\infty} x_j p_j t^j = L$$

exists, then we say that $x = (x_j)$ is convergent in the sense of power-series method [6, 16]. The power series method is regular if and only if

$$\lim_{t\to R^-} \frac{p_j t^j}{p(t)} = 0 \quad \text{for each} \quad j \in \mathbb{N}_0$$

(see [6]).

Let P_p be a regular power series method and let $E \subset \mathbb{N}_0$. If the limit

$$\delta_{P_p}(E) := \lim_{t\to R^-} \frac{1}{p(t)} \sum_{j\in E} p_j t^j$$

exists, then $\delta_{P_p}(E)$ is called the P_p-density of E.

The sequence $x = (x_j)$ of real numbers is said to be P_p-statistically convergent to L if, for every $\varepsilon > 0$, $\delta_{P_p}(E) = 0$, i.e., for every $\varepsilon > 0$,

$$\lim_{t\to R^-} \frac{1}{p(t)} \sum_{j\in E_\varepsilon} p_j t^j = 0,$$

where

$$E_\varepsilon = \left\{ j \in \mathbb{N}_0 \colon |x_j - L| \geq \varepsilon \right\}.$$

It is has been already shown that the statistical convergence and P_p-statistical convergence do not imply each other [18].

2. Korovkin Theorem in Weighted Spaces

In this section, we present a Korovkin-type approximation of a function f by means of positive linear operators from a weighted space C_{ϱ_1} into a weighted space B_{ϱ_2} with the use of P_p-statistical convergence. We are mainly motivated by the paper [4].

We first prove the following lemma required for the proof of our main theorem:

Lemma 1. *Let ϱ_1 and ϱ_2 be two weight functions satisfying (1). Assume that $\{T_n\}$ is a sequence of positive linear operators from C_{ϱ_1} into B_{ϱ_2}. If*

$$st_{P_p} - \lim \left\| T_n(F_v) - F_v \right\|_{\varrho_1} = 0, \tag{2}$$

where

$$F_v(x) = \frac{x^v \varrho_1(x)}{1+x^2}, \qquad v = 0, 1, 2,$$

then, for any $s > 0$ and all $f \in C_{\varrho_1}$,

$$st_{P_p} - \lim \left\| T_n(f) - f \right\|_{\varrho_2, [-s,s]} = 0.$$

Proof. By using the same procedure as in the proof of Lemma 2.3 in [4], we conclude that, for a given $f \in C_{\varrho_1}$, we have

$$\left\|T_n(f) - f\right\|_{\varrho_2,[-s,s]} \leq M\left(\varepsilon + \sum_{v=0}^{2}\left\|T_n(F_v) - F_v\right\|_{\varrho_1}\right)$$

for all $n \in \mathbb{N}$ and for some $M > 0$ independent of n and x. Further, for a given $r > 0$, we choose $\varepsilon > 0$ such that $M\varepsilon < r$. Thus, by setting

$$D = \left\{n \in \mathbb{N} \colon \sum_{v=0}^{2}\left\|T_n(F_v) - F_v\right\|_{\varrho_1} \geq \frac{r - M\varepsilon}{M}\right\},$$

$$D_0 = \left\{n \in \mathbb{N} \colon \left\|T_n(F_0) - F_0\right\|_{\varrho_1} \geq \frac{r - M\varepsilon}{3M}\right\},$$

$$D_1 = \left\{n \in \mathbb{N} \colon \left\|T_n(F_1) - F_1\right\|_{\varrho_1} \geq \frac{r - M\varepsilon}{3M}\right\},$$

$$D_2 = \left\{n \in \mathbb{N} \colon \left\|T_n(F_2) - F_2\right\|_{\varrho_1} \geq \frac{r - M\varepsilon}{3M}\right\},$$

it is easy to see that $D \subset D_0 \cup D_1 \cup D_2$. This implies that

$$0 \leq \delta_{P_p}\left(\left\{n \in \mathbb{N} \colon \left\|T_n(f) - f\right\|_{\varrho_2,[-s,s]} \geq r\right\}\right) \leq \sum_{i=0}^{2}\delta_{P_p}(D_i).$$

Thus, by using the hypothesis (2), we get

$$\delta_{P_p}\left(\left\{n \in \mathbb{N} \colon \left\|T_n(f) - f\right\|_{\varrho_2,[-s,s]} \geq r\right\}\right) = 0.$$

Lemma 1 is proved.

We can now formulate our main theorem, which can be regarded as a new approach to the Korovkin-type approximation in weighted spaces.

Theorem 2. *Let ϱ_1 and ϱ_2 be as in Lemma 1. Suppose that $\{T_n\}$ is a sequence of positive linear operators from C_{ϱ_1} into B_{ϱ_2}. Then, for all $f \in C_{\varrho_1}$, the following equality is true:*

$$st_{P_p} - \lim\left\|T_n(f) - f\right\|_{\varrho_2} = 0$$

provided that

$$st_{P_p} - \lim\left\|T_n(F_v) - F_v\right\|_{\varrho_1} = 0, \tag{3}$$

where

$$F_v(x) = \frac{x^v \varrho_1(x)}{1 + x^2}, \quad v = 0, 1, 2.$$

Proof. If we note that

$$T_n(F_v) = T_n(F_v) - F_v + F_v \qquad \text{and} \qquad F_v \in B_{\varrho_1}, \quad v = 0, 1, 2,$$

then it follows from (3) that $T_n(F_v) - F_v \in B_{\varrho_1}$. Hence, this gives $T_n(F_v) \in B_{\varrho_1}$, $v = 0, 1, 2$. Since $\varrho_1 = F_0 + F_2$, we get $T_n(\varrho_1) \in B_{\varrho_1}$. In addition, we have

$$\|T_n\|_{C_{\varrho_1} \to B_{\varrho_1}} = \|T_n(\varrho_1)\|_{\varrho_1} \leq M_1 < \infty.$$

This implies that $\{T_n\}$ is uniformly bounded from C_{ϱ_1} into B_{ϱ_1}. For a given $f \in C_{\varrho_1}$, we have

$$\|T_n(f)\|_{\varrho_1} \leq \|T_n\|_{C_{\varrho_1} \to B_{\varrho_1}} \|f\|_{\varrho_1} \leq M_1 |f|_{\varrho_1},$$

which implies that $T_n(f) \in B_{\varrho_1}$. Therefore, we obtain $T_n(C_{\varrho_1}) \subset B_{\varrho_1}$. Finally, in view of (1), (3) and the fact that $\|T_n\|_{C_{\varrho_1} \to B_{\varrho_2}} \leq M_2 < \infty$, the sequence $\{T_n\}$ is uniformly bounded from C_{ϱ_1} into B_{ϱ_2}.

Further, for a given $\varepsilon > 0$, we pick $s_0 > 0$ such that $\varrho_1(x) \leq \varepsilon \varrho_2(x)$ for all $|x| \geq s_0$. For $f \in C_{\varrho_1}$, we obtain

$$\begin{aligned} \left\|T_n(f) - f\right\|_{\varrho_2} &\leq \sup_{|x| \leq s_0} \frac{|T_n(f;x) - f(x)|}{\varrho_2(x)} + \sup_{|x| > s_0} \frac{|T_n(f;x) - f(x)|}{\varrho_2(x)} \\ &\leq \left\|T_n(f) - f\right\|_{\varrho_2, [-s_0, s_0]} + \varepsilon \left\|T_n(f) - f\right\|_{\varrho_1} \\ &\leq \left\|T_n(f) - f\right\|_{\varrho_2, [-s_0, s_0]} + \varepsilon \left(\|T_n(f)\|_{\varrho_1} + |f|_{\varrho_1}\right). \end{aligned}$$

Hence, by using Lemma 1, we complete the proof.

3. Rate of Convergence

Throughout this section, we assume that $\varrho_1(x) = 1 + x^2$ on $\mathbb{R}$. We obtain the rate of convergence by using the following weighted modulus of continuity:

$$\omega_{\varrho_1}(f;\delta) = \sup_{\substack{c|x-t| \leq \delta \\ x,t \in \mathbb{R}}} \frac{\left|f(x) - f(t)\right|}{\varrho_1(x) + \varrho_1(t)}, \quad \delta > 0, \quad f \in C_{\varrho_1}.$$

It can be easily seen that, for any $\delta > 0$,

$$\left|f(x) - f(t)\right| \leq \left[\varrho_1(x) + \varrho_1(t)\right] \left\{2 + \frac{|t-x|}{\delta}\right\} \omega_{\varrho_1}(f;\delta)$$

which implies that

$$\left|f(x) - f(t)\right| \leq 4\varrho_1(x).\varrho_1(t) \omega_{\varrho_1}(f;\delta) \left(1 + \frac{(t-x)^2}{\delta^2}\right).$$

By using similar operations, which have already been used in [4], for any $\delta > 0$ and all $f \in C_{\varrho_1}$, we can write

$$\begin{aligned}&\big|T_n\big(f(t);x\big) - f(x)\big|\\ &\qquad \le 4\varrho_1(x)\omega_{\varrho_1}(f;\delta)\left\{\big|T_n(\varrho_1(t);x) - \varrho_1(x)\big| + \varrho_1(x) + \frac{1}{\delta^2}T_n\left(\varrho_1(t)\phi_x(t);x\right)\right\}\\ &\qquad\qquad + |f(x)|\big|T_n(F_0(t);x) - F_0(x)\big|,\end{aligned}$$

where $\phi_x(t) = (t-x)^2$ and we get

$$\begin{aligned}&\big\|T_n(f) - f\big\|_{\varrho_2^2}\\ &\qquad \le 4\|\varrho_1\|_{\varrho_2}\omega_{\varrho_1}(f\,;\delta)\left\{\big\|T_n(\varrho_1) - \varrho_1\big\|_{\varrho_2} + \|\varrho_1\|_{\varrho_2} + \frac{1}{\delta^2}\big\|T_n(\varrho_1\phi_x)\big\|_{\varrho_2}\right\}\\ &\qquad\qquad + \|\varrho_1\|_{\varrho_2}\|f\|_{\varrho_2}\big\|T_n(F_0) - F_0\big\|_{\varrho_1}\end{aligned}$$

provided that $T_n(\varrho_1\phi_x) \in B_{\varrho_2}$. Thus, by considering $T_n\colon C_{\varrho_2} \to B_{\varrho_2}$ and assuming that $\varrho_1\phi_x \in C_{\varrho_2}$, we can see that $T_n(\varrho_1\phi_x) \in B_{\varrho_2}$. In this case, by setting

$$\delta := \delta_n = \sqrt{\big\|T_n(\varrho_1\phi_x)\big\|_{\varrho_2}}$$

and combining the inequalities presented above, we conclude that

$$\begin{aligned}\big\|T_n(f) - f\big\|_{\varrho_2^2} &\le 4\|\varrho_1\|_{\varrho_2}\omega_{\varrho_1}(f\,;\delta)\left\{\big\|T_n(\varrho_1) - \varrho_1\big\|_{\varrho_2} + \|\varrho_1\|_{\varrho_2} + 1\right\}\\ &\qquad + \|\varrho_1\|_{\varrho_2}\|f\|_{\varrho_2}\big\|T_n(F_0) - F_0\big\|_{\varrho_1}.\end{aligned}$$

We now introduce the P_p-statistical rate of convergence in the light of [7, 11].

Definition 1. *Let (a_n) be a positive nonincreasing sequence of real numbers and let P_p be a regular power-series method. A sequence $x = (x_n)$ is P_p-statistically convergent to the number L with a rate $o(a_n)$ if, for every $\varepsilon > 0$,*

$$\lim_{0<t\to R^-}\left[\frac{1}{p(t)}\sum_{n\,:\;|x_n - L| \ge \varepsilon a_n} p_n t^n\right] = 0.$$

In this case, $x_n - L = st_{P_p} - o(a_n)$ as $n \to \infty$.

We now find the P_p-statistical rate of convergence of the sequence $\{T_n\}$ in Theorem 2.

Conclusion 1. *Let ϱ_2 and T_n be as in Theorem 2 and let $T_n\left(\varrho_1(t-x)^2\right) \in B_{\varrho_2}$. Assume that (a_n), (b_n), and (c_n) are any nonincreasing sequences of positive real numbers. If the conditions* *(i)* $\big\|T_n(F_0) - F_0\big\|_{\varrho_1} = st_{P_p} - o(a_n),\ n \to \infty,$

(ii) $\big\|T_n(\varrho_1) - \varrho_1\big\|_{\varrho_2} = st_{P_p} - o(b_n),\ n \to \infty,$

(iii) $\omega_{\varrho_1}(f;\delta_n) = st_{P_p} - o(c_n),\ n \to \infty,$

are satisfied, then, for all $f \in C_{\varrho_1}$,

$$\big\|T_n(f) - f\big\|_{\varrho_2^2} = st_{P_p} - o(d_n), \quad n \to \infty,$$

where

$$\delta_n = \sqrt{\|T_n(\varrho_1\phi_x)\|_{\varrho_2}} \qquad \text{and} \qquad d_n = \max\big\{a_n, c_n, b_n c_n\big\}.$$

Proof. By using an idea similar to that used in Lemma 4 from [7] and taking care of the right-hand side of the following equality:

$$\big\|T_n(f) - f\big\|_{\varrho_2^2} \le 4\|\varrho_1\|_{\varrho_2}\omega_{\varrho_1}(f\,;\delta)\Big\{\big\|T_n(\varrho_1) - \varrho_1\big\|_{\varrho_2} + \|\varrho_1\|_{\varrho_2} + 1\Big\}$$

$$+ \|\varrho_1\|_{\varrho_2}\|f\|_{\varrho_2}\big\|T_n(F_0) - F_0\big\|_{\varrho_1},$$

we can easily see that

$$d_n = \max\{a_n, c_n, c_n b_n\}.$$

4. Applications

In this section, we provide an example of a sequence of positive linear operators illustrating that Theorem 2 is stronger than Theorem 1.

Example 1. Consider

$$p_n = \begin{cases} 1, & n = 2k, \\ 0, & n = 2k+1, \end{cases} \qquad s_n = \begin{cases} 0, & n = 2k, \\ 1, & n = 2k+1. \end{cases}$$

It is easy to see that P_p is a regular power-series method. For every $\varepsilon > 0,$ since

$$E_\varepsilon = \big\{n \in \mathbb{N}_0 \colon |s_n - 0| \ge \varepsilon\big\} = \big\{n \in \mathbb{N}_0 \colon n = 2k+1\big\},$$

we have

$$\delta_{P_p}(E_\varepsilon) = 0,$$

i.e., the sequence (s_n) is P_p-statistically convergent to 0. Let $\varrho_1(x) = 1 + x^2$ and let $\varrho_2(x) = 1 + x^4$. In this case, the test functions F_v become $F_v(x) = x^v$. Consider the following Gauss–Weierstrass operators defined by

$$W_n\big(f(t);x\big) = \sqrt{\frac{n}{2\pi}} \int\limits_{-\infty}^{+\infty} f(t)e^{-\frac{1}{2}n(t-x)^2}\,dt.$$

We observe that, for each $n \in \mathbb{N},\ W_n$ is a positive linear operator acting from C_{ϱ_1} into B_{ϱ_2}. We also get

$$W_n\left(F_0(t);x\right)=1, \qquad W_n\left(F_1(t);x\right)=x, \qquad W_n\left(F_2(t);x\right)=x^2+\frac{1}{n}.$$

Furthermore, since

$$W_n\big(\varrho_1(t);x\big)=x^2+1+\frac{1}{n}\leq x^2+2\leq 2\varrho_1(x)\leq 4\varrho_2(x),$$

$\{W_n\}$ is a uniformly bounded sequence of positive linear operators from C_{ϱ_1} into B_{ϱ_1} (or into B_{ϱ_2}). We define

$$T_n(f(t);x)=(1+s_n)W_n(f(t);x).$$

Thus, we can immediately show that $\{T_n\}$ is a uniformly bounded sequence of positive linear operators by its construction. By using the test functions

$$F_v(x)=x^v, \quad v=0,1,2,$$

we obtain

$$T_n(F_0(t);x)=1+s_n,$$

$$T_n(F_1(t);x)=(1+s_n)x,$$

$$T_n(F_2(t);x)=(1+s_n)\left(x^2+\frac{1}{n}\right),$$

which implies that the sequence $\{T_n\}$ satisfies the conditions of the main theorem and it is possible to conclude that, for all $f \in C_{\varrho_1}$,

$$st_{P_p}-\lim\left\|T_n(f)-f\right\|_{\varrho_2}=0.$$

However, since (s_n) is not convergent to zero and not statistically convergent, $\{T_n\}$ satisfies neither Theorem 1, nor the statistical version.

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